

Multi-band optical imaging

From fusion to change detection

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Joint work with Q. Wei, V. Ferraris, J.-Y. Tournéret and M. Chabert

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Multi-band optical imaging

Multi/hyper-spectral images

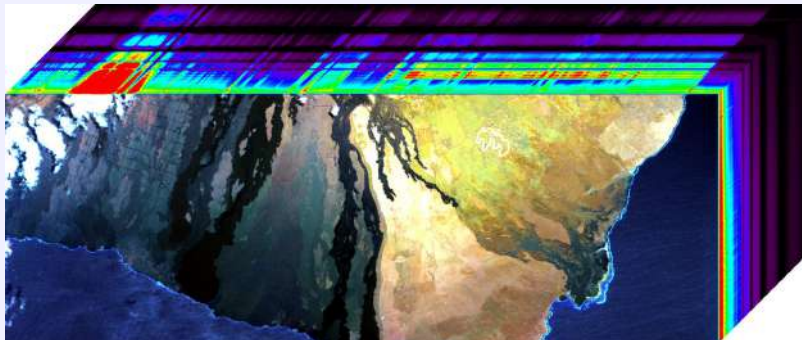
- same scene observed at different wavelengths

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Hyperspectral Cube



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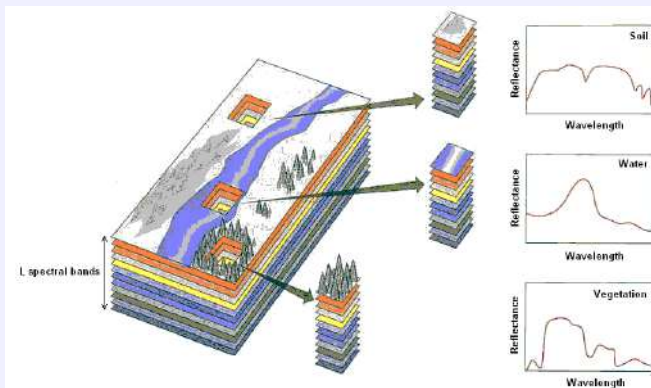
- same scene observed at different wavelengths,
- pixel represented by a vector of tens/hundreds of measurements.

Multi-band optical imaging

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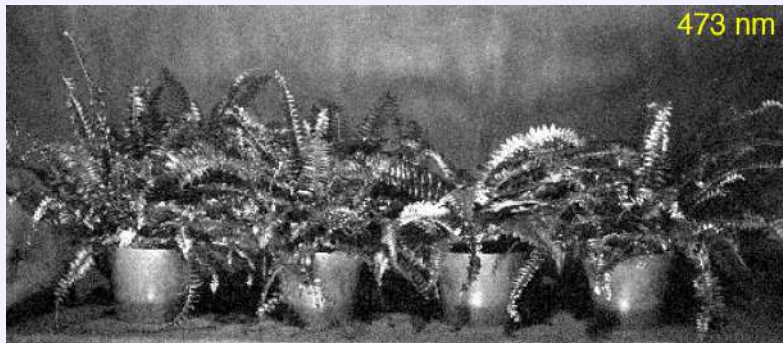
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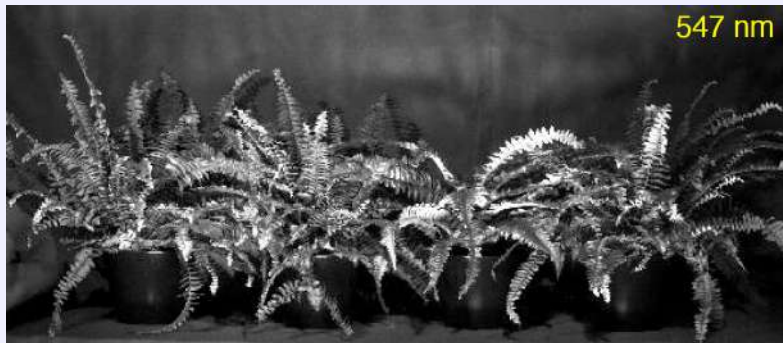
Multi-band optical imaging



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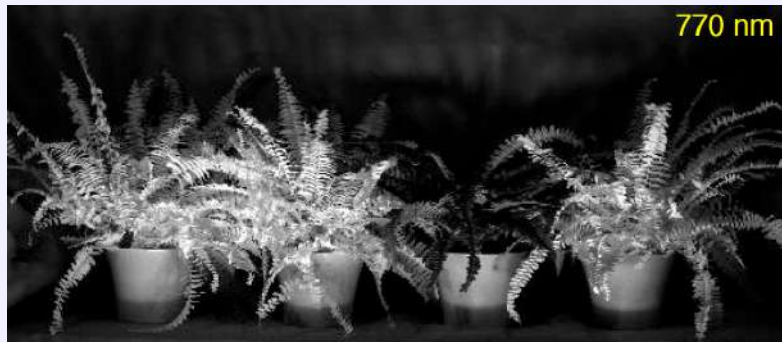
Multi-band optical imaging



Multi-band optical imaging



Multi-band optical imaging



Multi-band optical imaging

Spatial vs. spectral resolution trade-off

Panchromatic images (PAN)

- no spectral resolution (only 1 band),
- very high spatial resolution ($\sim 10\text{cm}$).

Multispectral images (MS)

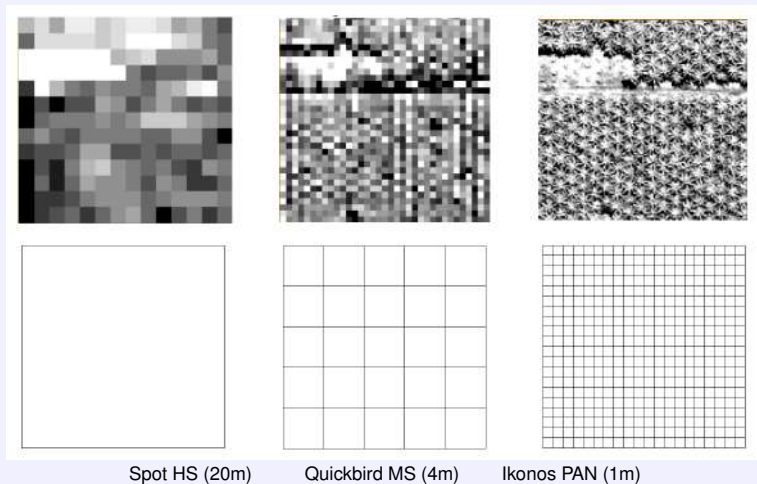
- low spectral resolution (~ 10 bands),
- high spatial resolution ($\sim 1\text{m}$).

Hyperspectral images (HS)

- high spectral resolution (~ 100 bands),
- low spatial resolution ($\sim 10\text{m}$).

Multi-band optical imaging

Spatial vs. spectral resolution trade-off



Outline

Introduction

Multi-band optical image fusion

- Problem statement

- Fast fusion solving a Sylvester equation

- Experiments

Multi-band optical image change detection

- Fusion approach

- Robust Fusion approach

Conclusions

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Multi-band optical image fusion

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- Fast fusion solving a Sylvester equation

 - From maximum likelihood estimator...

 - ... to maximum a posteriori estimators

- Experiments

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- Fusion approach

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Conclusions

Multiple image fusion

Pansharpening: PAN+MS fusion

- incorporate the spatial details of the PAN image into the MS image
- huge literature
- main approaches rely on band substitution

Hyperspectral pansharpening: PAN+HS fusion

- incorporate the spatial details of the PAN image into the HS image
- more difficult due to the size of the HS image
- specific methods should be developed

Multi-band image fusion: MS+HS fusion

- incorporate the spatial details of the MS image into the HS image
- more difficult since the spatial details contained in a multi-band image
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Problem statement

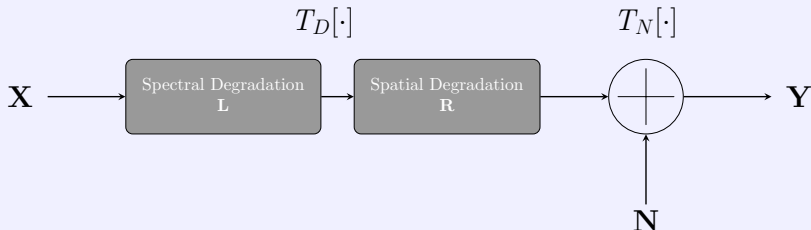


(a) Hyperspectral Image (size: $99 \times 46 \times 224$, res.: $20\text{m} \times 20\text{m}$) (b) Multispectral Image (size: $396 \times 184 \times 4$ res.: $5\text{m} \times 5\text{m}$) (c) Target (size: $396 \times 184 \times 224$ res.: $5\text{m} \times 5\text{m}$)

Name	AVIRIS (HS)	SPOT-5 (MS)	Pleiades (MS)	WorldView-3 (MS)
Res. (m)	20	10	2	1.24
# bands	224	4	4	8

Table: Some existing remote sensors characteristics

Forward model for multi-band optical images



$$\mathbf{Y} = \mathbf{L}\mathbf{X}\mathbf{R} + \mathbf{N}$$

where

- $\mathbf{Y}$ observed multiband image of low spatial and/or spectral resolutions
(row $\leftrightarrow$ band, column $\leftrightarrow$ pixel)
- $\mathbf{X}$ (unknown) latent image of high spatial and/or spectral resolutions
(row $\leftrightarrow$ band, column $\leftrightarrow$ pixel)
- $\mathbf{L}$ spectral degradation matrix
- $\mathbf{R}$ spatial degradation matrix, e.g., decomposed as $\mathbf{R} = \mathbf{B}\mathbf{S}$ with
 - $\mathbf{B}$ spatial blur
 - $\mathbf{S}$ spatial subsampling

Complementary acquisitions: forward models

- $\mathbf{X} \in \mathbb{R}^{m_\lambda \times n}$: full resolution unknown image



X

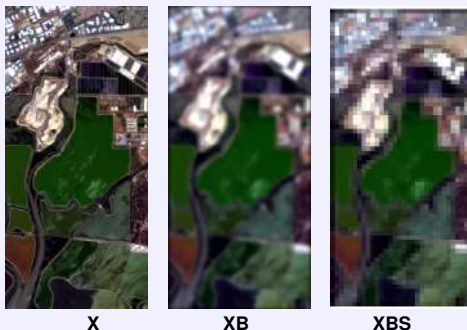
Complementary acquisitions: forward models

- $\mathbf{X} \in \mathbb{R}^{m_\lambda \times n}$: full resolution unknown image
- $\mathbf{B} \in \mathbb{R}^{n \times n}$: cyclic convolution operator acting on the bands



Complementary acquisitions: forward models

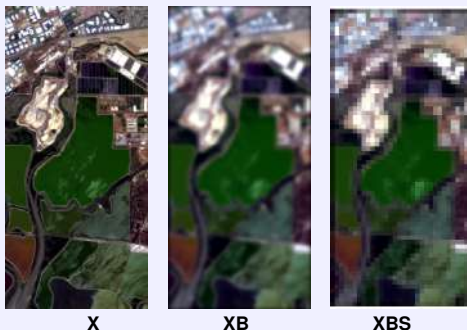
- $\mathbf{X} \in \mathbb{R}^{m \times n}$: full resolution unknown image
- $\mathbf{B} \in \mathbb{R}^{n \times n}$: cyclic convolution operator acting on the bands
- $\mathbf{S} \in \mathbb{R}^{n \times m}$: downsampling operator



Complementary acquisitions: forward models

$$\mathbf{Y}_H \approx \mathbf{XBS}$$

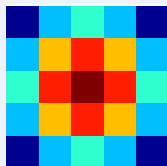
- $\mathbf{X} \in \mathbb{R}^{m_\lambda \times n}$: full resolution unknown image
- $\mathbf{Y}_H \in \mathbb{R}^{m_\lambda \times m}$: observed HS image
- $\mathbf{B} \in \mathbb{R}^{n \times n}$: **cyclic convolution operator** acting on the bands
- $\mathbf{S} \in \mathbb{R}^{n \times m}$: **downsampling** operator



Complementary acquisitions: forward models

$$\mathbf{Y}_H \approx \mathbf{XBS},$$

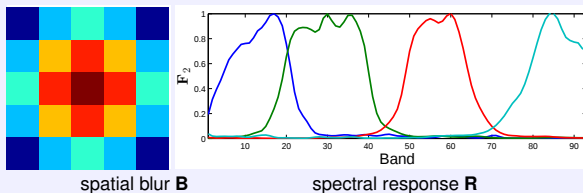
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spatial blur $\mathbf{B}$

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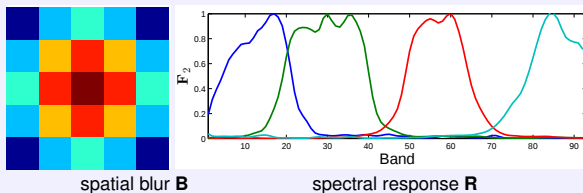
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- $\mathbf{S} \in \mathbb{R}^{n \times m}$: downsampling operator
- $\mathbf{R} \in \mathbb{R}^{n_\lambda \times m_\lambda}$: spectral response of the MS sensor



Complementary acquisitions: forward models

$$\mathbf{Y}_H \approx \mathbf{XBS} \quad , \quad \mathbf{Y}_M \approx \mathbf{RX}$$

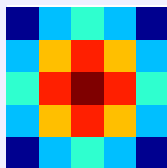
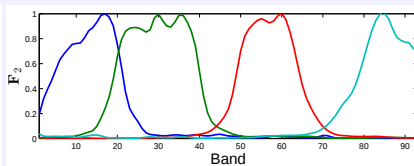
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- $\mathbf{B} \in \mathbb{R}^{n \times n}$: cyclic convolution operator acting on the bands
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- $\mathbf{R} \in \mathbb{R}^{n_\lambda \times m_\lambda}$: spectral response of the MS sensor



Complementary acquisitions: forward models

$$\mathbf{Y}_H = \mathbf{XBS} + \mathbf{N}_H, \quad \mathbf{Y}_M = \mathbf{RX} + \mathbf{N}_M$$

- $\mathbf{X} \in \mathbb{R}^{m_\lambda \times n}$: full resolution unknown image
- $\mathbf{Y}_H \in \mathbb{R}^{m_\lambda \times m}$: observed HS image
- $\mathbf{Y}_M \in \mathbb{R}^{n_\lambda \times n}$: observed MS image
- $\mathbf{B} \in \mathbb{R}^{n \times n}$: cyclic convolution operator acting on the bands
- $\mathbf{S} \in \mathbb{R}^{n \times m}$: downsampling operator
- $\mathbf{R} \in \mathbb{R}^{n_\lambda \times m_\lambda}$: spectral response of the MS sensor
- $\mathbf{N}_H \in \mathbb{R}^{m_\lambda \times m}$ and $\mathbf{N}_M \in \mathbb{R}^{n_\lambda \times n}$: HS and MS noises

spatial blur $\mathbf{B}$ spectral response $\mathbf{R}$

Noise statistics

Gaussian assumption

$$\begin{aligned}\mathbf{N}_H | \boldsymbol{\Lambda}_H &\sim \mathcal{MN}_{m_\lambda, m}(\mathbf{0}_{m_\lambda, m}, \boldsymbol{\Lambda}_H, \mathbf{I}_m) \\ \mathbf{N}_M | \boldsymbol{\Lambda}_M &\sim \mathcal{MN}_{n_\lambda, n}(\mathbf{0}_{n_\lambda, n}, \boldsymbol{\Lambda}_M, \mathbf{I}_n)\end{aligned}$$

where

- $\boldsymbol{\Lambda}_H = \text{diag} \left\{ s_{H,1}^2, \dots, s_{H,m_\lambda}^2 \right\}$ (hyperspectral noise variances)
- $\boldsymbol{\Lambda}_M = \text{diag} \left\{ s_{M,1}^2, \dots, s_{M,n_\lambda}^2 \right\}$ (multispectral noise variances)

and the pdf of a matrix normal distribution is defined by

$$p(\mathbf{Z} | \bar{\mathbf{Z}}, \boldsymbol{\Sigma}_r, \boldsymbol{\Sigma}_c) \propto \exp \left(-\frac{1}{2} \text{tr} \left[\boldsymbol{\Sigma}_c^{-1} (\mathbf{Z} - \bar{\mathbf{Z}})^T \boldsymbol{\Sigma}_r^{-1} (\mathbf{Z} - \bar{\mathbf{Z}}) \right] \right)$$

→ band-dependent noise

→ pixel-independent noise

Likelihood of the observations

Given the forward model (characterized by both left- and right-operators)

$$\begin{aligned}\mathbf{Y}_H &= \mathbf{XBS} + \mathbf{N}_H \\ \mathbf{Y}_M &= \mathbf{RX} + \mathbf{N}_M\end{aligned}$$

the two likelihood functions express as

$$\begin{aligned}\mathbf{Y}_H|\mathbf{X} &\sim \mathcal{MN}_{m_\lambda, m}(\mathbf{XBS}, \mathbf{\Lambda}_H, \mathbf{I}_m) \\ \mathbf{Y}_M|\mathbf{X} &\sim \mathcal{MN}_{n_\lambda, n}(\mathbf{RX}, \mathbf{\Lambda}_M, \mathbf{I}_n)\end{aligned}$$

Joint likelihood

HS and MS images acquired by distinct sensors

- independent HS and MS noises
- independent observed images, cond. on $\mathbf{X}$

$$f(\mathbf{Y}_H, \mathbf{Y}_M|\mathbf{X}) = f(\mathbf{Y}_H|\mathbf{X}) f(\mathbf{Y}_M|\mathbf{X})$$

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Multi-band image fusion as an estimation problem

Maximum likelihood estimation

Maximizing the two data-fitting terms writes

$$\hat{\mathbf{X}} \in \underset{\mathbf{X}}{\operatorname{argmin}} -\log f(\mathbf{Y}_H|\mathbf{X}) - \log f(\mathbf{Y}_M|\mathbf{X})$$

formulated as a weighted least-square regression

$$\hat{\mathbf{X}} \in \underset{\mathbf{X}}{\operatorname{argmin}} \|\mathbf{Y}_M - \mathbf{R}\mathbf{X}\|_{\Lambda_M^{-1}}^2 + \|\mathbf{Y}_H - \mathbf{X}\mathbf{B}\mathbf{S}\|_{\Lambda_H^{-1}}^2$$

Main issues

- (generally) large scale problem
- (generally) ill-posed (at least ill-conditioned) problem

Regularization required...

- (always) in the spectral domain
- (optional) in the spatial domain

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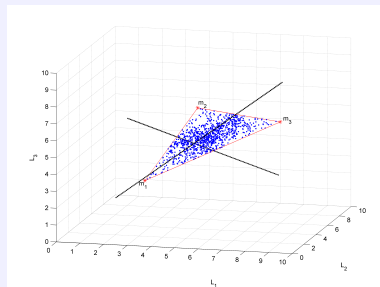
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Spectral regularization

Low-rank representation

Hyperspectral pixels live in a (much) lower-dimensional subspace...



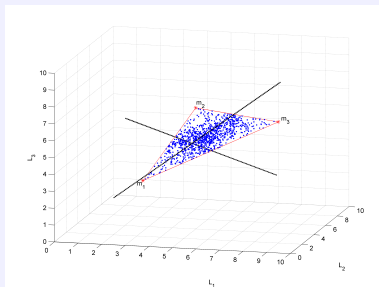
Unknown image $\mathbf{X}$ enforced to be decomposed as

$$\mathbf{X} = \mathbf{H}\mathbf{U}$$

i.e., its pixels live in a lower-dimensional subspace ($\mathbb{R}^{\tilde{m}_\lambda}$ with $\tilde{m}_\lambda \ll m_\lambda$) spanned by the columns of $\mathbf{H} \in \mathbb{R}^{m_\lambda \times \tilde{m}_\lambda}$ (estimated or known a priori).

Spectral regularization Low-rank representation

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Optimization problem

Given the spectral regularization, the optimization problem writes

$$\hat{\mathbf{U}} \in \underset{\mathbf{U}}{\operatorname{argmin}} \mathcal{J}(\mathbf{U})$$

with

$$\mathcal{J}(\mathbf{U}) = \left\| \boldsymbol{\Lambda}_{\mathbf{H}}^{-1} (\mathbf{Y}_{\mathbf{H}} - \mathbf{H}\mathbf{U}\mathbf{B}\mathbf{S}) \right\|_{\mathbf{F}}^2 + \left\| \boldsymbol{\Lambda}_{\mathbf{M}}^{-1} (\mathbf{Y}_{\mathbf{M}} - \mathbf{R}\mathbf{H}\mathbf{U}) \right\|_{\mathbf{F}}^2$$

$$\nabla \mathcal{J}(\mathbf{U}) = 0$$

$$\Leftrightarrow$$

finding $\mathbf{U}$ such that $\mathbf{C}_1 \mathbf{U} + \mathbf{U} \mathbf{C}_2 = \mathbf{C}_3$

with

$$\mathbf{C}_1 = \left[\mathbf{H}^H \boldsymbol{\Lambda}_{\mathbf{H}}^{-1} \mathbf{H} \right]^{-1} \left[(\mathbf{R}\mathbf{H})^H \boldsymbol{\Lambda}_{\mathbf{M}}^{-1} (\mathbf{R}\mathbf{H}) \right]$$

$$\mathbf{C}_2 = \left[\mathbf{B}\mathbf{S} (\mathbf{B}\mathbf{S})^H \right]$$

$$\mathbf{C}_3 = \text{term depending on } \mathbf{Y}_{\mathbf{H}} \text{ and } \mathbf{Y}_{\mathbf{M}} \text{ (ind. on } \mathbf{U})$$

Solving a Sylvester matrix equation

$$\mathbf{C}_1 \mathbf{U} + \mathbf{U} \mathbf{C}_2 = \mathbf{C}_3$$

Main issue

$\mathbf{C}_2 = \mathbf{B}\mathbf{S}(\mathbf{B}\mathbf{S})^H$ is not diagonalizable!

In the literature...

- *general resolution*: Bartels-Stewart algorithm (1972), with complexity of $\mathcal{O}(n^3)$
→ impossible in practice
- *in the context of fusion*: iterative algos, e.g., gradient descent, ADMM...
→ time consuming

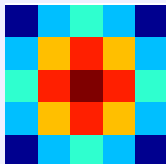
Our contribution

We showed that an explicit solution can be written and easily computed! (see [WDT15a, WDT⁺16])

Remark: result can be applied to (because generalizes) superresolution [ZWB⁺16]...

Assumption 1

The blurring matrix $\mathbf{B}$ is a **block circulant matrix with circulant blocks (BCCB)**.

*Assumption 2*

The decimation matrix $\mathbf{S}$ corresponds to **downsampling** the original signal and its conjugate transpose $\mathbf{S}^H$ **interpolates** the decimated signal **with zeros**.

e.g.

$$\mathbf{S} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Fast fUtion based on a Sylvester Equation (FUSE)

Input: $\mathbf{Y}_M, \mathbf{Y}_H, \mathbf{\Lambda}_M, \mathbf{\Lambda}_H, \mathbf{R}, \mathbf{B}, \mathbf{S}, \mathbf{H}$
 $\mathbf{D} \leftarrow \mathbf{F}^H \mathbf{B} \mathbf{F}$ and $\underline{\mathbf{D}} \leftarrow \mathbf{D}^* \mathbf{D}$ /* Circulant matrix: $\mathbf{B} = \mathbf{F} \mathbf{D} \mathbf{F}^H$ */ $\mathbf{C}_1 \leftarrow (\mathbf{H}^H \mathbf{\Lambda}_H^{-1} \mathbf{H})^{-1} ((\mathbf{R} \mathbf{H})^H \mathbf{\Lambda}_L^{-1} \mathbf{R} \mathbf{H})$ /* Compute $\mathbf{C}_1$ */ $(\mathbf{Q}, \mathbf{\Lambda}_C) \leftarrow \text{EigDec}(\mathbf{C}_1)$ /* Eigen-dec of $\mathbf{C}_1$: $\mathbf{C}_1 = \mathbf{Q} \mathbf{\Lambda}_C \mathbf{Q}^{-1}$ */ $\bar{\mathbf{C}}_3 \leftarrow \mathbf{Q}^{-1} (\mathbf{H}^H \mathbf{\Lambda}_H^{-1} \mathbf{H})^{-1} (\mathbf{H}^H \mathbf{\Lambda}_H^{-1} \mathbf{Y}_H (\mathbf{B} \mathbf{S})^H + (\mathbf{R} \mathbf{H})^H \mathbf{\Lambda}_L^{-1} \mathbf{Y}_M) \mathbf{B} \mathbf{F} \mathbf{P}^{-1}$ **for** $l = 1$ to $\tilde{m}_\lambda$ **do**

$$\underline{\mathbf{u}}_{l,1} = (\bar{\mathbf{C}}_3)_{l,1} \left(\frac{1}{d} \sum_{i=1}^d \underline{\mathbf{D}}_i + \lambda_C^l \mathbf{I}_n \right)^{-1}$$
for $j = 2$ to d **do**

$$\underline{\mathbf{u}}_{l,j} = \frac{1}{\lambda_C^l} \left((\bar{\mathbf{C}}_3)_{l,j} - \frac{1}{d} \underline{\mathbf{u}}_{l,1} \underline{\mathbf{D}}_j \right)$$
end**end** $\hat{\mathbf{U}} = \mathbf{Q} \underline{\mathbf{U}} \mathbf{P} \mathbf{D}^{-1} \mathbf{F}^H$ **Output:** $\hat{\mathbf{X}} = \mathbf{H} \hat{\mathbf{U}}$

From ML to MAP estimators

Incorporating (spatial) regularization

$$\hat{\mathbf{U}} \in \underset{\mathbf{U}}{\operatorname{argmin}} \left\| \mathbf{\Lambda}_H^{-1} (\mathbf{Y}_H - \mathbf{HUBS}) \right\|_F^2 + \left\| \mathbf{\Lambda}_M^{-1} (\mathbf{Y}_M - \mathbf{RHU}) \right\|_F^2 + \mu \phi(\mathbf{U})$$

where

- $\phi(\mathbf{U}) = \left\| \mathbf{\Gamma} (\mathbf{U} - \bar{\mathbf{U}}) \right\|_F^2$: (generalized) Thikonov regularizations
 - $\mathbf{\Gamma} = \mathbf{I}$ and $\bar{\mathbf{U}}$ = “crude estimate”: supervised naive Gaussian prior [WDT15b]
→ closed-form solution
 - joint estimation of $\underline{\mathbf{\Lambda}} = \{\mathbf{\Lambda}_H, \mathbf{\Lambda}_M\}$ and μ : unsupervised naive Gaussian prior [WDT15c]
→ closed-form solution embedded in BCD algorithm

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Gaussian prior

Unsupervised naïve Gaussian prior: closed-form solution embedded in BCD (FUSE-BCD)

Input: $\mathbf{Y}_H, \mathbf{Y}_M, \tilde{m}_\lambda, \mathbf{B}, \mathbf{S}, \mathbf{R}, \mathbf{H}$

for $t = 1$ **to** T **do**

// Optimize w.r.t. to $\mathbf{U}$

$\mathbf{U}_t = \arg \min_{\mathbf{U}} L(\mathbf{U}, \underline{\Lambda}_{t-1}, \mu_{t-1})$

/ Sylvester equation */*

// Optimize w.r.t. $\underline{\Lambda}$

$\underline{\Lambda}_t = \arg \min_{\underline{\Lambda}} L(\mathbf{U}_t, \underline{\Lambda}, \mu_{t-1})$

// Optimize w.r.t. μ

$\mu_t = \arg \min_{\mu} L(\mathbf{U}_t, \underline{\Lambda}_t, \mu)$

end

$\hat{\mathbf{U}} \leftarrow \mathbf{U}_T$

Output: $\hat{\mathbf{X}} = \mathbf{H}\hat{\mathbf{U}}$

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 - joint estimation of $\underline{\mathbf{\Lambda}} = \{\mathbf{\Lambda}_H, \mathbf{\Lambda}_M\}$ and μ : unsupervised naive Gaussian prior [WDT15c]
→ closed-form solution embedded in BCD algorithm
- $\phi(\mathbf{U}) = \left\| \mathbf{U} - \mathbf{DA} \right\|_F^2$: sparse representation based on dictionary learning [WBDT15]
 - $\mathbf{D}$: dictionary learnt beforehand
 - $\mathbf{A}$: code estimated (with sparse support learnt beforehand)
→ closed-form solution embedded in BCD algorithm

From ML to MAP estimators

Incorporating (spatial) regularization

$$\hat{\mathbf{U}} \in \underset{\mathbf{U}}{\operatorname{argmin}} \left\| \mathbf{\Lambda}_H^{-1} (\mathbf{Y}_H - \mathbf{HUBS}) \right\|_F^2 + \left\| \mathbf{\Lambda}_M^{-1} (\mathbf{Y}_M - \mathbf{RHU}) \right\|_F^2 + \mu \phi(\mathbf{U})$$

where

- $\phi(\mathbf{U}) = \left\| \mathbf{\Gamma} (\mathbf{U} - \bar{\mathbf{U}}) \right\|_F^2$: (generalized) Thikonov regularizations
 - $\mathbf{\Gamma} = \mathbf{I}$ and $\bar{\mathbf{U}}$ = “crude estimate”: supervised naive Gaussian prior [WDT15b]
 - closed-form solution
 - joint estimation of $\underline{\mathbf{\Lambda}} = \{\mathbf{\Lambda}_H, \mathbf{\Lambda}_M\}$ and μ : unsupervised naive Gaussian prior [WDT15c]
 - closed-form solution embedded in BCD algorithm
- $\phi(\mathbf{U}) = \left\| \mathbf{U} - \mathbf{DA} \right\|_F^2$: sparse representation based on dictionary learning [WBDT15]
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 - $\mathbf{A}$: code estimated (with sparse support learnt beforehand)
 - closed-form solution embedded in BCD algorithm

Sparse representation

Sparse prior: closed-form solution embedded in BCD (FUSE-BCD)

Input: $\mathbf{Y}_H, \mathbf{Y}_M, \tilde{m}_\lambda, \mathbf{B}, \mathbf{S}, \mathbf{R}, \mathbf{H}, \text{SNR}_H, \text{SNR}_M, n_{\max}$

// Rough estimation of $\mathbf{U}$

Approximate $\bar{\mathbf{U}}$ using $\mathbf{Y}_M$ and $\mathbf{Y}_H$;

// Online dictionary learning

$\hat{\mathbf{D}} \leftarrow \text{ODL}(\bar{\mathbf{U}})$;

// Sparse coding

$\hat{\mathbf{A}} \leftarrow \text{OMP}(\hat{\mathbf{D}}, \bar{\mathbf{U}}, n_{\max})$;

// Computing support

$\hat{\Omega} \leftarrow \hat{\mathbf{A}} \neq 0$;

// Start alternate optimization

for $t = 1$ **to** T **do**

// Optimize w.r.t. to $\mathbf{U}$

$\mathbf{U}_t = \arg \min_{\mathbf{U}} L(\mathbf{U}, \mathbf{A}_{t-1})$;

/ Sylvester equation */*

// Optimize w.r.t. to $\mathbf{A}$

$\mathbf{A}_t = \arg \min_{\mathbf{A}} L(\mathbf{U}_t, \mathbf{A})$;

/ solved with LS */*

end

Output: $\hat{\mathbf{X}} = \mathbf{H}\hat{\mathbf{U}}$

From ML to MAP estimators

Incorporating (spatial) regularization

$$\hat{\mathbf{U}} \in \underset{\mathbf{U}}{\operatorname{argmin}} \left\| \mathbf{\Lambda}_H^{-1} (\mathbf{Y}_H - \mathbf{HUBS}) \right\|_F^2 + \left\| \mathbf{\Lambda}_M^{-1} (\mathbf{Y}_M - \mathbf{RHU}) \right\|_F^2 + \mu \phi(\mathbf{U})$$

where

- $\phi(\mathbf{U}) = \left\| \mathbf{\Gamma} (\mathbf{U} - \bar{\mathbf{U}}) \right\|_F^2$: (generalized) Thikonov regularizations
 - $\mathbf{\Gamma} = \mathbf{I}$ and $\bar{\mathbf{U}}$ = “crude estimate”: supervised naive Gaussian prior [WDT15b]
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 - $\mathbf{D}$: dictionary learnt beforehand
 - $\mathbf{A}$: code estimated (with sparse support learnt beforehand)
 - closed-form solution embedded in BCD algorithm
- $\phi(\mathbf{U}) = \sum_{j=1}^{\tilde{m}} \lambda \operatorname{TV} [\mathbf{U}_{j,:}]$: band-wise total variation [SoBAC15]
 - closed-form solution embedded in ADMM algorithm

From ML to MAP estimators

Incorporating (spatial) regularization

$$\hat{\mathbf{U}} \in \underset{\mathbf{U}}{\operatorname{argmin}} \left\| \mathbf{\Lambda}_H^{-1} (\mathbf{Y}_H - \mathbf{HUBS}) \right\|_F^2 + \left\| \mathbf{\Lambda}_M^{-1} (\mathbf{Y}_M - \mathbf{RHU}) \right\|_F^2 + \mu \phi(\mathbf{U})$$

where

- $\phi(\mathbf{U}) = \left\| \mathbf{\Gamma} (\mathbf{U} - \bar{\mathbf{U}}) \right\|_F^2$: (generalized) Thikonov regularizations
 - $\mathbf{\Gamma} = \mathbf{I}$ and $\bar{\mathbf{U}}$ = “crude estimate”: supervised naive Gaussian prior [WDT15b]
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 - $\mathbf{D}$: dictionary learnt beforehand
 - $\mathbf{A}$: code estimated (with sparse support learnt beforehand)
 - closed-form solution embedded in BCD algorithm
- $\phi(\mathbf{U}) = \sum_{j=1}^{\tilde{m}} \lambda \operatorname{TV} [\mathbf{U}_{j,:}]$: band-wise total variation [SoBAC15]
 - closed-form solution embedded in ADMM algorithm

Non-Gaussian prior

Non-Gaussian prior, such as TV

$$\arg \min_{\mathbf{U}} \underbrace{\frac{1}{2} \|\mathbf{\Lambda}_H^{-\frac{1}{2}} (\mathbf{Y}_H - \mathbf{HUBS})\|_F^2}_{\text{HS data term}} + \underbrace{\frac{1}{2} \|\mathbf{\Lambda}_M^{-\frac{1}{2}} (\mathbf{Y}_M - \mathbf{RHU})\|_F^2}_{\text{MS data term}} + \underbrace{\lambda \text{TV}(\mathbf{U})}_{\text{regularizer}}.$$

can be equivalently solved as:

$$\arg \min_{\mathbf{U}, \mathbf{V}} \frac{1}{2} \|\mathbf{\Lambda}_H^{-\frac{1}{2}} (\mathbf{Y}_H - \mathbf{HUBS})\|_F^2 + \frac{1}{2} \|\mathbf{\Lambda}_M^{-\frac{1}{2}} (\mathbf{Y}_M - \mathbf{RHU})\|_F^2 + \lambda \text{TV}(\mathbf{V}) \text{ s.t. } \mathbf{U} = \mathbf{V}$$

- ADMM algorithm: alternate minimization (FUSE-ADMM)
 - closed-form solution of the Sylvester equation
 - proximal mapping

Illustrative results

PAN + HS fusion / Gaussian prior



(left to right) HS image, PAN image, ground truth, ADMM, proposed method.

Performance and computational times

HS + MS fusion / various regularizations

Table: RSNR (in dB), UIQI, SAM (in degree), ERGAS, DD (in 10^{-3}) and time (in second).

Regularization	Methods	RSNR	UIQI	SAM	ERGAS	DD	Time
supervised naïve Gaussian	ADMM	29.321	0.9906	1.555	0.888	7.115	126.83
	FUSE	29.372	0.9908	1.551	0.879	7.092	0.38
unsupervised naïve Gaussian	ADMM-BCD	29.084	0.9902	1.615	0.913	7.341	99.55
	FUSE-BCD	29.077	0.9902	1.623	0.913	7.368	1.09
sparse representation	ADMM-BCD	29.582	0.9911	1.423	0.872	6.678	162.88
	FUSE-BCD	29.688	0.9913	1.431	0.856	6.672	73.66
TV	ADMM	29.473	0.9912	1.503	0.861	6.922	134.21
	FUSE-ADMM	29.631	0.9915	1.477	0.845	6.788	90.99

- The computational time is **decreased significantly!**

Comparison with state-of-the-art methods

PAN + HS fusion

Table: Characteristics of the three datasets [LBDB⁺15]

dataset	dimensions	spatial res	N	instrument
Moffett	PAN 185×395 HS 37×79	20m 100m	224	AVIRIS
Camargue	PAN 500×500 HS 100×100	4m 20m	125	HyMap
Garons	PAN 400×400 HS 80×80	4m 20m	125	HyMap

Comparison with state-of-the-art methods

PAN + HS fusion

Table: Quality measures for the Moffett field dataset

method	CC	SAM	RMSE	ERGAS	Time(sec)
SFIM	0.92955	9.5271	365.2577	6.5429	1.26
MTF-GLP	0.93919	9.4599	352.1290	6.0491	1.86
MTF-GLP-HPM	0.93817	9.3567	354.8167	6.1992	1.71
GS	0.90521	14.1636	443.4351	7.5952	4.77
GSA	0.93857	11.2758	363.7090	6.2359	5.52
PCA	0.89580	14.6132	463.2204	7.9283	3.46
GFPCA	0.91614	11.3363	404.2979	7.0619	2.58
CNMF	0.95496	9.4177	314.4632	5.4200	10.98
Supervised Gaussian	0.97785	7.1308	220.0310	3.7807	1.31
Sparse represent.	0.98168	6.6392	200.3365	3.4281	133.61
HySure	0.97059	7.6351	254.2005	4.3582	140.05

Comparison with state-of-the-art methods

PAN + HS fusion

Table: Quality measures for the Camargue dataset

method	CC	SAM	RMSE	ERGAS	Time(sec)
SFIM	0.91886	4.2895	637.1451	3.4159	3.47
MTF-GLP	0.92397	4.3378	622.4711	3.2666	4.26
MTF-GLP-HPM	0.92599	4.2821	611.9161	3.2497	4.25
GS	0.91262	4.4982	665.0173	3.5490	8.29
GSA	0.92826	4.1950	587.1322	3.1940	8.73
PCA	0.90350	5.1637	710.3275	3.8680	8.92
GFPCA	0.89042	4.8472	745.6006	4.0001	8.51
CNMF	0.93000	4.4187	591.3190	3.1762	47.54
Supervised Gaussian	0.95195	3.6428	489.5634	2.6286	7.35
Sparse represent.	0.95882	3.3345	448.1721	2.4712	485.13
HySure	0.94650	3.8767	511.8525	2.8181	296.27

Comparison with state-of-the-art methods

PAN + HS fusion

Table: Quality measures for the Garons dataset

method	CC	SAM	RMSE	ERGAS	Time(sec)
SFIM	0.77052	6.7356	1036.4695	5.1702	2.74
MTF-GLP	0.80124	6.6155	956.3047	4.8245	4.00
MTF-GLP-HPM	0.79989	6.6905	962.1076	4.8280	2.98
GS	0.80347	6.6627	1037.6446	5.1373	5.56
GSA	0.80717	6.7719	928.6229	4.7076	5.99
PCA	0.81452	6.6343	1021.8547	5.0166	6.09
GFPCA	0.63390	7.4415	1312.0373	6.3416	4.36
CNMF	0.82993	6.9522	893.9194	4.4927	23.98
Supervised Gaussian	0.86857	5.8749	784.1298	3.9147	3.07
Sparse represent.	0.87834	5.6377	750.3510	3.7629	259.44
HySure	0.86080	6.0224	778.1051	4.0454	177.60

Outline

Introduction

Multi-band optical image fusion

- Problem statement

- Fast fusion solving a Sylvester equation

- Experiments

Multi-band optical image change detection

- Fusion approach

 - Problem statement

 - Resolution pipeline

 - Experiments

- Robust Fusion approach

 - Problem statement

 - Algorithm

 - Experimental results

Conclusions

Change Detection (CD)

Input

- Two or more multitemporal images.
- Same geographical spot (scene).

Output

- Change map.



Source: RafaelRabellodeBarros

CD in remote sensing context

Applications:

- Land-use and land-cover analysis.
- Urban monitoring.
- Environmental surveillance.
- Defense and security.

Taxonomy of methods:

- According to supervision.
 - Supervised.
 - Unsupervised.
- According to modality.
 - Same modality.
 - Multimodality.

Supervised vs. Unsupervised

Supervised

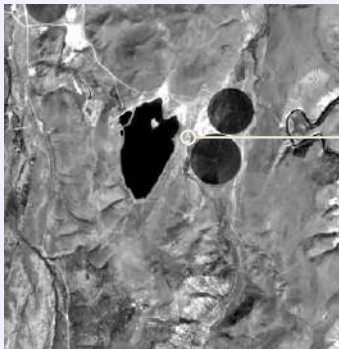
- Require ground information.
- More appropriate to multimodal images.
- Higher complexity of methods.
- Good overall performance.
- Depend on training set.
- Less appealing for real applications.

Unsupervised

- Does not require any ground information.
- Generally applied to same modalities.
- Lower complexity of methods.
- Lower overall performance.
- Generally require preprocessing steps.
- Automatic behaviour.

Favorable scenario

Landsat 8 04/15/2015 (PAN - 15m)



Landsat 8 09/22/2015 (PAN - 15m)



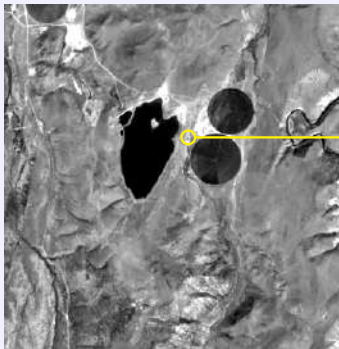
- Same modality.
- Identical resolutions.

Comparison of homologous pixels!

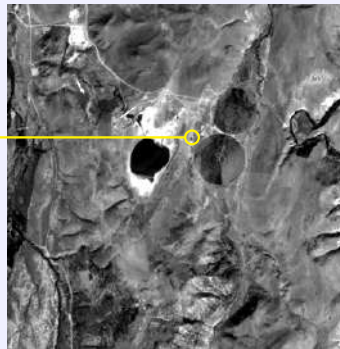
Same modality CD!

Favorable scenario

Landsat 8 04/15/2015 (PAN - 15m)



Landsat 8 09/22/2015 (PAN - 15m)



- Same modality.
- Identical resolutions.

→ Comparison of homologous pixels!

Same modality CD!

Unfavorable scenario

Emergence situation:

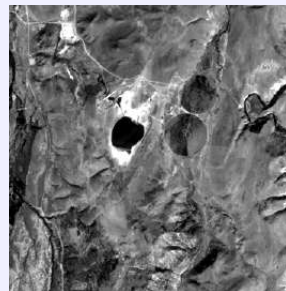
- Natural disaster.
- Punctual missions.
- Defense and security.

Need:

- Multimodal CD.



Landsat 8 04/15/2015 (MS - 30m)



Landsat 8 09/22/2015 (PAN - 15m)

State-of-the-art

Principle:

- Identical resolutions obtained through independent and individual transformation over the considered images. [KCS⁺13]
- Multimodality CD achieved by supervised or semi-supervised methods [PCP⁺15].

Methods:

- Worst-case (WC): Degradation of both observed images.
- Degradation-Superresolution (DS): Spectral degradation followed by spatial Superresolution of LR-HS observed image.
- Superresolution-Degradation (SD): Spatial Superresolution followed by spectral degradation of LR-HS observed image.
- Coupled dictionary learning [GZSL16].

Problems:

- No joint processing.
- Loss of information (DS and SD).
- Loss of resolution (WC).

Adopted strategy: leveraging on fusion

General principle:

- Consider two images of same region acquired at same time (no change).
- Fused image evidences information contained in the pair of input images.
- Able to deal with different resolutions (e.g. pansharpening).

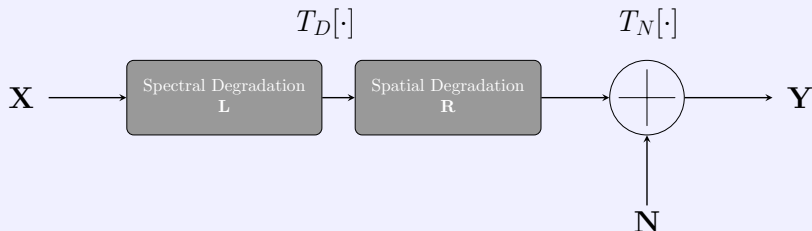
Today:

- Fusion-based approach [FDWC18]
- Robust-fusion based approach [FDWC17]

Remarks:

- Easier to manipulate optical images due to the noise statistics.
- 85% of the total earth observation satellites are optical [oCS17].

Forward model for multi-band optical images



$$\mathbf{Y} = \mathbf{LXR} + \mathbf{N}$$

where

- $\mathbf{Y}$ observed multiband image of low spatial and/or spectral resolutions
(row $\leftrightarrow$ band, column $\leftrightarrow$ pixel)
- $\mathbf{X}$ (unknown) latent image of high spatial and/or spectral resolutions
(row $\leftrightarrow$ band, column $\leftrightarrow$ pixel)
- $\mathbf{L}$ spectral degradation matrix
- $\mathbf{R}$ spatial degradation matrix, e.g., decomposed as $\mathbf{R} = \mathbf{BS}$ with
 - $\mathbf{B}$ spatial blur
 - $\mathbf{S}$ spatial subsampling

Applicative scenarios

	Forward model #1		Forward model #2		Comments
	Spectral degradation	Spatial degradation	Spectral degradation	Spatial degradation	
S_1	—	—	—	—	Conventional CD framework – Y_1 and Y_2 of same spatial and spectral resolutions
S_2	L_1	—	—	—	Y_1 of lower spectral resolution Y_1 and Y_2 of same spatial resolutions
S_3	—	R_1	—	—	Y_1 of lower spatial resolution Y_1 and Y_2 of same spectral resolutions
S_4	—	R_1	L_2	—	Y_1 and Y_2 of complementary resolutions
S_5	L_1	R_1	—	—	Y_1 of low spatial and spectral resolutions
S_6	—	R_1	—	R_2	Generalization of S_3 with non-integer relative spatial downsampling factor
S_7	L_1	R_1	—	R_2	Generalization of S_4 with non-integer relative spatial downsampling factor
S_8	L_1	—	L_2	—	Generalization of S_2 with some non-overlapping spectral bands
S_9	L_1	R_1	L_2	—	Generalization of S_4 with some non-overlapping spectral bands
S_{10}	L_1	R_1	L_2	R_2	Generalization of S_4 with some non-overlapping spectral bands and non-integer relative spatial downsampling factor

Table: Overview of the spectral and spatial degradations w.r.t. experimental scenarios. The symbol — stands for “no degradation” [FDC20].

Applicative scenarios

	Forward model #1		Forward model #2		Comments
	Spectral degradation	Spatial degradation	Spectral degradation	Spatial degradation	
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S_4	—	R_1	L_2	—	Y_1 and Y_2 of complementary resolutions
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S_6	—	R_1	—	R_2	Generalization of S_3 with non-integer relative spatial downsampling factor
S_7	L_1	R_1	—	R_2	Generalization of S_4 with non-integer relative spatial downsampling factor
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Table: Overview of the spectral and spatial degradations w.r.t. experimental scenarios. The symbol — stands for “no degradation” [FDC20].

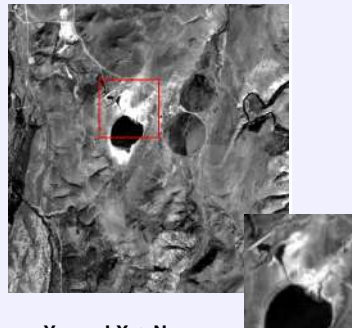
Scenario S_4

Landsat 8 04/15/2015 (MS - 30m)



$$\mathbf{Y}_{\text{LR}} = \mathbf{X}\mathbf{R} + \mathbf{N}_{\text{LR}}$$

Landsat 8 09/22/2015 (PAN - 15m)



$$\mathbf{Y}_{\text{HR}} = \mathbf{L}\mathbf{X} + \mathbf{N}_{\text{HR}}$$

- Time ordering independent: $t_1 \neq t_2$.

Problem statement

Joint observation model

$$\mathbf{Y}_{\text{LR}} = \mathbf{X}\mathbf{R} + \mathbf{N}_{\text{LR}}$$

$$\mathbf{Y}_{\text{HR}} = \mathbf{L}\mathbf{X} + \mathbf{N}_{\text{HR}}$$

Fusion process

$$\hat{\mathbf{X}} \leftarrow \text{FUSION}(\mathbf{Y}_{\text{LR}}, \mathbf{Y}_{\text{HR}})$$

"Predicted" pseudo-observed images

$$\begin{aligned}\hat{\mathbf{Y}}_{\text{LR}} &\triangleq \hat{\mathbf{X}}\mathbf{R} \\ \hat{\mathbf{Y}}_{\text{HR}} &\triangleq \mathbf{L}\hat{\mathbf{X}}\end{aligned}$$

Fusion properties [LdBD⁺ 15, WRM97]

- *Synthesis*: fused image $\approx$ image obtained by the sensor of the target resolution.
- *Consistency*: reversibility of the fusion process.

Consistency-based CD hypothesis testing

$$\mathcal{H}_0 : \begin{cases} \mathbf{Y}_{\text{LR}} = \hat{\mathbf{Y}}_{\text{LR}} \\ \mathbf{Y}_{\text{HR}} = \hat{\mathbf{Y}}_{\text{HR}} \end{cases} \quad (\text{no change})$$

$$\mathcal{H}_1 : \begin{cases} \mathbf{Y}_{\text{LR}} \neq \hat{\mathbf{Y}}_{\text{LR}} \\ \mathbf{Y}_{\text{HR}} \neq \hat{\mathbf{Y}}_{\text{HR}} \end{cases} \quad (\text{change})$$

Problem statement

Joint observation model

$$\mathbf{Y}_{\text{LR}} = \mathbf{X}\mathbf{R} + \mathbf{N}_{\text{LR}}$$

$$\mathbf{Y}_{\text{HR}} = \mathbf{L}\mathbf{X} + \mathbf{N}_{\text{HR}}$$

Fusion process

$$\hat{\mathbf{X}} \leftarrow \text{FUSION}(\mathbf{Y}_{\text{LR}}, \mathbf{Y}_{\text{HR}})$$

"Predicted" pseudo-observed images

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Problem statement

Joint observation model

$$\mathbf{Y}_{\text{LR}} = \mathbf{X}\mathbf{R} + \mathbf{N}_{\text{LR}}$$

$$\mathbf{Y}_{\text{HR}} = \mathbf{L}\mathbf{X} + \mathbf{N}_{\text{HR}}$$

Fusion process

$$\hat{\mathbf{X}} \leftarrow \text{FUSION}(\mathbf{Y}_{\text{LR}}, \mathbf{Y}_{\text{HR}})$$

"Predicted" pseudo-observed images

$$\begin{aligned} \hat{\mathbf{Y}}_{\text{LR}} &\triangleq \hat{\mathbf{X}}\mathbf{R} \\ \hat{\mathbf{Y}}_{\text{HR}} &\triangleq \mathbf{L}\hat{\mathbf{X}} \end{aligned}$$

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$$\mathcal{H}_1 : \begin{cases} \mathbf{Y}_{\text{LR}} \neq \hat{\mathbf{Y}}_{\text{LR}} \\ \mathbf{Y}_{\text{HR}} \neq \hat{\mathbf{Y}}_{\text{HR}} \end{cases} \quad (\text{change})$$

Problem statement

Joint observation model

$$\mathbf{Y}_{\text{LR}} = \mathbf{X}\mathbf{R} + \mathbf{N}_{\text{LR}}$$

$$\mathbf{Y}_{\text{HR}} = \mathbf{L}\mathbf{X} + \mathbf{N}_{\text{HR}}$$

Fusion process

$$\hat{\mathbf{X}} \leftarrow \text{FUSION}(\mathbf{Y}_{\text{LR}}, \mathbf{Y}_{\text{HR}})$$

"Predicted" pseudo-observed images

$$\begin{aligned} \hat{\mathbf{Y}}_{\text{LR}} &\triangleq \hat{\mathbf{X}}\mathbf{R} \\ \hat{\mathbf{Y}}_{\text{HR}} &\triangleq \mathbf{L}\hat{\mathbf{X}} \end{aligned}$$

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Consistency-based CD hypothesis testing

$$\mathcal{H}_0 : \begin{cases} \mathbf{Y}_{\text{LR}} = \hat{\mathbf{Y}}_{\text{LR}} \\ \mathbf{Y}_{\text{HR}} = \hat{\mathbf{Y}}_{\text{HR}} \end{cases} \quad (\text{no change})$$

$$\mathcal{H}_1 : \begin{cases} \mathbf{Y}_{\text{LR}} \neq \hat{\mathbf{Y}}_{\text{LR}} \\ \mathbf{Y}_{\text{HR}} \neq \hat{\mathbf{Y}}_{\text{HR}} \end{cases} \quad (\text{change})$$

- 1 **fusion**: estimating $\hat{\mathbf{X}}$ from $\mathbf{Y}_{\text{LR}}$ and $\mathbf{Y}_{\text{HR}}$
 - Tailored by the end user [FYDC18]
e.g., **Fast fusion** based on solving a SE

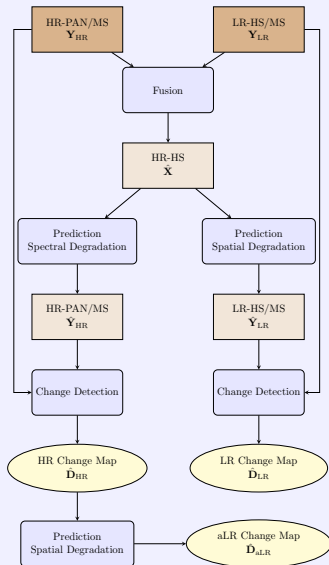
- 2** *prediction*: reconstructing $\hat{\mathbf{Y}}_{\text{LR}}$ and $\hat{\mathbf{Y}}_{\text{HR}}$ from $\hat{\mathbf{X}}$

$$\begin{aligned}\hat{\mathbf{Y}}_{\text{LR}} &= \hat{\mathbf{X}}\mathbf{R} \\ \hat{\mathbf{Y}}_{\text{HR}} &= \mathbf{L}\hat{\mathbf{X}}.\end{aligned}$$

- 3** *decision*: deriving change maps $\hat{\mathbf{D}}_{\text{LR}}$ and $\hat{\mathbf{D}}_{\text{HR}}$ from, resp.,

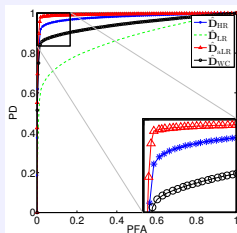
$$\begin{aligned}\Upsilon_{\text{LR}} &= \left\{ \mathbf{Y}_{\text{LR}}, \hat{\mathbf{Y}}_{\text{LR}} \right\}, \\ \Upsilon_{\text{HR}} &= \left\{ \mathbf{Y}_{\text{HR}}, \hat{\mathbf{Y}}_{\text{HR}} \right\}.\end{aligned}$$

- Tailored by the end user.
e.g., Change Vector Analysis (CVA) [BB07]

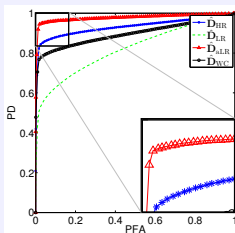


Experiments on synthetic images

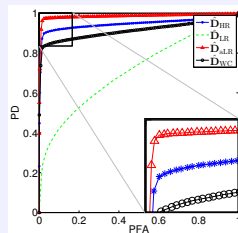
Detection performance



(a) Situation 1: HR-MS/LR-HS



(b) Situation 2: HR-PAN/LR-HS



(c) Situation 3: HR-PAN/LR-MS

Table: Situations 1, 2 & 3: quantitative detection performance (AUC and distance).

		$\hat{D}_{HR}$	$\hat{D}_{LR}$	$\hat{D}_{aLR}$	$\hat{D}_{WC}$
Situation 1	AUC	0.981039	0.867478	0.992242	0.941408
	Dist.	0.951995	0.789379	0.979298	0.887789
Situation 2	AUC	0.931047	0.819679	0.977362	0.89517
	Dist.	0.883488	0.737274	0.952995	0.833783
Situation 3	AUC	0.94522	0.711167	0.984833	0.911311
	Dist.	0.915992	0.647865	0.972997	0.864686

Experiments on real images

Data description

Observed image at t_1 (04/15/2015):

- Local: Lake-Tahoe (CA) USA.
- Sensor: Landsat 8.
- Image size: 175×180 pixels.
- Spatial resolution: 30m per pixel.
- Spectral resolution: 3 spectral bands (MS) in RGB visible spectrum.

Observed image at t_2 (09/22/2015):

- Local: Lake-Tahoe (CA) USA.
- Sensor: Landsat 8.
- Image size: 350×360 pixels.
- Spatial resolution: 15m per pixel.
- Spectral resolution: PAN in RGB visible spectrum.

Preprocessing:

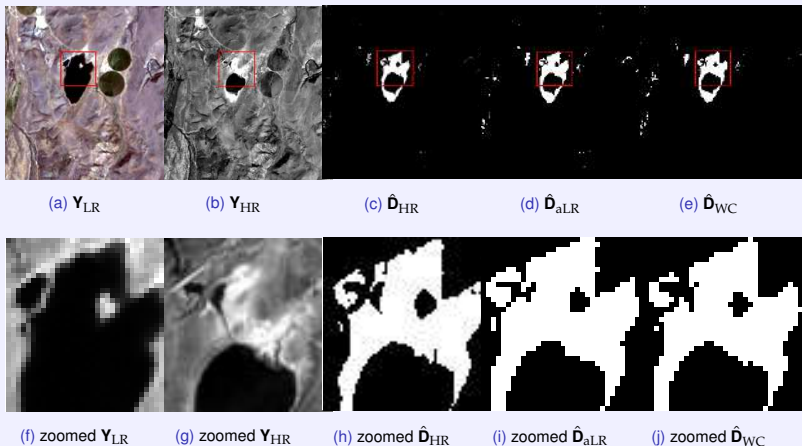
- Manual alignment.

Compared Methods:

- Fusion approach ($\hat{\mathbf{D}}_{\text{HR}}, \hat{\mathbf{D}}_{\text{aLR}}$).
- Worst-case approach ($\hat{\mathbf{D}}_{\text{WC}}$).

Experiments on real images

Visual results



Real scenario (LR-MS and HR-PAN): (a) LR-MS observed image $\mathbf{Y}_{LR}$, (b) HR-PAN observed image $\mathbf{Y}_{HR}$, (c) change mask $\hat{\mathbf{D}}_{HR}$, (d) change mask $\hat{\mathbf{D}}_{aLR}$, (e) change mask $\hat{\mathbf{D}}_{WC}$ estimated by the worst-case approach. From (f) to (j): zoomed versions of the regions delineated in red in (a)–(e).

Problem statement

Joint observation model: from fusion...

$$\mathbf{Y}_{\text{LR}} = \mathbf{X} \mathbf{R} + \mathbf{N}_{\text{LR}}$$

$$\mathbf{Y}_{\text{HR}} = \mathbf{L} \mathbf{X} + \mathbf{N}_{\text{HR}}$$

with

■ $\mathbf{X}_1$: latent image at t_1

■ $\mathbf{X}_2$: latent image at t_2

or, equivalently, $\mathbf{X}_2 = \mathbf{X}_1 + \Delta \mathbf{X}$ with

■ $\Delta \mathbf{X}$: change image

$$\Delta \mathbf{X} = [\Delta \mathbf{x}_1, \dots, \Delta \mathbf{x}_n] \text{ and } \Delta \mathbf{x}_i = [\Delta x_{1,i}, \dots, \Delta x_{m_\lambda,i}]^T$$

CD hypothesis testing

Decision rule for the i th pixel ($i = 1, \dots, n$)

$$\mathcal{H}_0 : \|\Delta \mathbf{x}_i\|_2 < \tau \quad (\text{no change})$$

$$\mathcal{H}_1 : \|\Delta \mathbf{x}_i\|_2 \geq \tau \quad (\text{change})$$

Problem statement

*Joint observation model: from fusion... to **robust** fusion*

$$\mathbf{Y}_{\text{LR}} = \mathbf{X}_1 \mathbf{R} + \mathbf{N}_{\text{LR}}$$

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$$\mathcal{H}_0 : \|\Delta \mathbf{x}_i\|_2 < \tau \quad (\text{no change})$$

$$\mathcal{H}_1 : \|\Delta \mathbf{x}_i\|_2 \geq \tau \quad (\text{change})$$

Optimization problem

Likelihood of the observations

$$\mathbf{Y}_{\text{LR}}|\mathbf{X}_1 \sim \mathcal{MN}_{n_{\lambda}, m_1}(\mathbf{X}_1 \mathbf{R}, \mathbf{\Lambda}_{\text{LR}}, \mathbf{I}_{m_1})$$

$$\mathbf{Y}_{\text{HR}}|\mathbf{X}_2 \sim \mathcal{MN}_{m_{\lambda_2}, n}(\mathbf{L}\mathbf{X}_2, \mathbf{\Lambda}_{\text{HR}}, \mathbf{I}_n)$$

Maximum a posteriori (MAP) estimator

$$\begin{aligned} \left\{ \hat{\mathbf{X}}_{1, \text{MAP}}, \Delta \hat{\mathbf{X}}_{\text{MAP}} \right\} \in \arg \min_{\mathbf{X}_1, \Delta \mathbf{X}} & \left\| \mathbf{\Lambda}_{\text{LR}}^{-\frac{1}{2}} (\mathbf{Y}_{\text{LR}} - \mathbf{X}_1 \mathbf{R}) \right\|_F^2 \\ & + \left\| \mathbf{\Lambda}_{\text{HR}}^{-\frac{1}{2}} (\mathbf{Y}_{\text{HR}} - \mathbf{L}(\mathbf{X}_1 + \Delta \mathbf{X})) \right\|_F^2 \\ & + \mu \phi_1(\mathbf{X}_1) + \gamma \phi_2(\Delta \mathbf{X}) \end{aligned}$$

Key ingredient: ϕ_2

Spatial sparsity of the changes through a group-lasso regularization

$$\begin{aligned} \phi_2(\Delta \mathbf{X}) &= \|\Delta \mathbf{X}\|_{2,1} \\ &= \sum_{i=1}^n \|\Delta \mathbf{x}_i\|_2 = \|\mathbf{e}\|_1 \end{aligned}$$

promoting a sparse change image energy vector $\mathbf{e} = [\|\Delta \mathbf{x}_1\|_2, \dots, \|\Delta \mathbf{x}_n\|_2]$.

Optimization problem

Likelihood of the observations

$$\mathbf{Y}_{\text{LR}} | \mathbf{X}_1 \sim \mathcal{MN}_{n_{\lambda}, m_1}(\mathbf{X}_1 \mathbf{R}, \mathbf{\Lambda}_{\text{LR}}, \mathbf{I}_{m_1})$$

$$\mathbf{Y}_{\text{HR}} | \mathbf{X}_2 \sim \mathcal{MN}_{m_{\lambda_2}, n}(\mathbf{L} \mathbf{X}_2, \mathbf{\Lambda}_{\text{HR}}, \mathbf{I}_n)$$

Maximum a posteriori (MAP) estimator

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promoting a sparse change image energy vector $\mathbf{e} = [\|\Delta \mathbf{x}_1\|_2, \dots, \|\Delta \mathbf{x}_n\|_2]$.

Solution

Robust fusion: alternate minimization algorithm [FDWC17, FDC20]

Data: $\mathbf{Y}_{\text{HR}}, \mathbf{Y}_{\text{LR}}, \mathbf{L}, \mathbf{R}$

Input: $\Delta \mathbf{X}^0$

for $k = 1, \dots, K$ **do**

// Fusion step

$$\mathbf{X}_1^{(k+1)} = \arg \min_{\mathbf{X}_1} \mathcal{J}(\mathbf{X}_1, \Delta \mathbf{X}^{(k)})$$

// Correction step

$$\Delta \mathbf{X}^{(k+1)} = \arg \min_{\Delta \mathbf{X}} \mathcal{J}(\mathbf{X}_1^{(k+1)}, \Delta \mathbf{X})$$

end

$$\hat{\mathbf{X}}_{1,\text{MAP}} \triangleq \mathbf{X}_1^{(K+1)} \text{ and } \Delta \hat{\mathbf{X}}_{\text{MAP}} \triangleq \Delta \hat{\mathbf{X}}^{(K+1)}$$

Output: $\hat{\mathbf{X}}_{1,\text{MAP}}, \Delta \hat{\mathbf{X}}_{\text{MAP}}$

Characteristics

- Iterative minimization,
- Problem split into 2 simple sub-problems,
- Convergence guarantee.

Optimization w.r.t. $\mathbf{X}_1$ (fixing $\Delta\mathbf{X} = \Delta\mathbf{X}^{(k)}$)

$$\min_{\mathbf{X}_1, \Delta\mathbf{X}} \left\| \Lambda_{\text{LR}}^{-\frac{1}{2}} (\mathbf{Y}_{\text{LR}} - \mathbf{X}_1 \mathbf{R}) \right\|_{\text{F}}^2 + \left\| \Lambda_{\text{HR}}^{-\frac{1}{2}} (\mathbf{Y}_{\text{HR}} - \mathbf{L}(\mathbf{X}_1 + \Delta\mathbf{X})) \right\|_{\text{F}}^2 + \mu \|\mathbf{X}_1\|_{\text{F}}^2 + \gamma \|\Delta\mathbf{X}\|_{2,1}$$

Optimization problem

$$\mathbf{X}_1^{(k+1)} = \arg \min_{\mathbf{X}_1} \left\| \Lambda_{\text{LR}}^{-\frac{1}{2}} (\mathbf{Y}_{\text{LR}} - \mathbf{X}_1 \mathbf{R}) \right\|_{\text{F}}^2 + \left\| \Lambda_{\text{HR}}^{-\frac{1}{2}} (\mathbf{Y}_{\text{HR}} - \mathbf{L}(\mathbf{X}_1 + \Delta\mathbf{X}^{(k)})) \right\|_{\text{F}}^2 + \mu \|\mathbf{X}_1\|_{\text{F}}^2$$

rewritten as

$$\mathbf{X}_1^{(k+1)} = \arg \min_{\mathbf{X}_1} \left\| \Lambda_{\text{LR}}^{-\frac{1}{2}} (\mathbf{Y}_{\text{LR}} - \mathbf{X}_1 \mathbf{R}) \right\|_{\text{F}}^2 + \left\| \Lambda_{\text{HR}}^{-\frac{1}{2}} (\tilde{\mathbf{Y}}_{\text{HR}}^{(k)} - \mathbf{L}\mathbf{X}_1) \right\|_{\text{F}}^2 + \mu \|\mathbf{X}_1\|_{\text{F}}^2$$

with

- $\tilde{\mathbf{Y}}_{\text{HR}}^k = \mathbf{Y}_{\text{HR}} - \mathbf{L}\Delta\mathbf{X}^{(k)}$: pseudo-observed image at t_1 .
- Equivalent to an **image fusion** problem (single latent image estimation $\mathbf{X}_1$).
- Two quadratic data-fitting terms + Thikonov regularization
→ fast and explicit solution based on solving a **Sylvester equation**.

Optimization w.r.t. $\mathbf{X}_1$ (fixing $\Delta\mathbf{X} = \Delta\mathbf{X}^{(k)}$)

$$\min_{\mathbf{X}_1, \Delta\mathbf{X}} \left\| \mathbf{\Lambda}_{\text{LR}}^{-\frac{1}{2}} (\mathbf{Y}_{\text{LR}} - \mathbf{X}_1 \mathbf{R}) \right\|_{\text{F}}^2 + \left\| \mathbf{\Lambda}_{\text{HR}}^{-\frac{1}{2}} (\mathbf{Y}_{\text{HR}} - \mathbf{L}(\mathbf{X}_1 + \Delta\mathbf{X})) \right\|_{\text{F}}^2 + \mu \|\mathbf{X}_1\|_{\text{F}}^2 + \gamma \|\Delta\mathbf{X}\|_{2,1}$$

Optimization problem

$$\mathbf{X}_1^{(k+1)} = \arg \min_{\mathbf{X}_1} \left\| \mathbf{\Lambda}_{\text{LR}}^{-\frac{1}{2}} (\mathbf{Y}_{\text{LR}} - \mathbf{X}_1 \mathbf{R}) \right\|_{\text{F}}^2 + \left\| \mathbf{\Lambda}_{\text{HR}}^{-\frac{1}{2}} (\mathbf{Y}_{\text{HR}} - \mathbf{L}(\mathbf{X}_1 + \Delta\mathbf{X}^{(k)})) \right\|_{\text{F}}^2 + \mu \|\mathbf{X}_1\|_{\text{F}}^2$$

rewritten as

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with

- $\tilde{\mathbf{Y}}_{\text{HR}}^k = \mathbf{Y}_{\text{HR}} - \mathbf{L}\Delta\mathbf{X}^{(k)}$: pseudo-observed image at t_1 .
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with

- $\tilde{\mathbf{Y}}_{\text{HR}}^k = \mathbf{Y}_{\text{HR}} - \mathbf{L}\Delta\mathbf{X}^{(k)}$: pseudo-observed image at t_1 .
- Equivalent to an **image fusion** problem (single latent image estimation $\mathbf{X}_1$).
- Two quadratic data-fitting terms + Thikonov regularization
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Optimization w.r.t. $\Delta \mathbf{X}$ (fixing $\mathbf{X}_1 = \mathbf{X}_1^{(k)}$)

$$\min_{\mathbf{X}_1, \Delta \mathbf{X}} \left\| \Lambda_{\text{LR}}^{-\frac{1}{2}} (\mathbf{Y}_{\text{LR}} - \mathbf{X}_1 \mathbf{R}) \right\|_{\text{F}}^2 + \left\| \Lambda_{\text{HR}}^{-\frac{1}{2}} (\mathbf{Y}_{\text{HR}} - \mathbf{L}(\mathbf{X}_1 + \Delta \mathbf{X})) \right\|_{\text{F}}^2 + \mu \|\mathbf{X}_1\|_{\text{F}}^2 + \gamma \|\Delta \mathbf{X}\|_{2,1}$$

Optimization problem

$$\Delta \mathbf{X}^{(k+1)} = \arg \min_{\Delta \mathbf{X}} \left\| \Lambda_{\text{HR}}^{-\frac{1}{2}} \left(\Delta \mathbf{Y}_{\text{HR}} - \mathbf{L}(\mathbf{X}_1^k + \Delta \mathbf{X}) \right) \right\|_{\text{F}}^2 + \gamma \|\Delta \mathbf{X}\|_{2,1}$$

rewritten as

$$\Delta \mathbf{X}^{(k+1)} = \arg \min_{\Delta \mathbf{X}} \left\| \Lambda_{\text{HR}}^{-\frac{1}{2}} \left(\Delta \tilde{\mathbf{Y}}_{\text{HR}}^k - \mathbf{L} \Delta \mathbf{X} \right) \right\|_{\text{F}}^2 + \gamma \|\Delta \mathbf{X}\|_{2,1}$$

with

- $\Delta \tilde{\mathbf{Y}}_{\text{HR}}^k = \mathbf{Y}_{\text{HR}} - \mathbf{L} \mathbf{X}_1^k$: predicted change image at t_2 .
- Equivalent to a **spectral deblurring** problem.
- Convex data-fitting term and regularization, non-smooth regularization
→ solution using proximal algorithms (i.e. forward-backward).

Optimization w.r.t. $\Delta \mathbf{X}$ (fixing $\mathbf{X}_1 = \mathbf{X}_1^{(k)}$)

$$\min_{\mathbf{X}_1, \Delta \mathbf{X}} \left\| \Lambda_{\text{LR}}^{-\frac{1}{2}} (\mathbf{Y}_{\text{LR}} - \mathbf{X}_1 \mathbf{R}) \right\|_{\text{F}}^2 + \left\| \Lambda_{\text{HR}}^{-\frac{1}{2}} (\mathbf{Y}_{\text{HR}} - \mathbf{L}(\mathbf{X}_1 + \Delta \mathbf{X})) \right\|_{\text{F}}^2 + \mu \|\mathbf{X}_1\|_{\text{F}}^2 + \gamma \|\Delta \mathbf{X}\|_{2,1}$$

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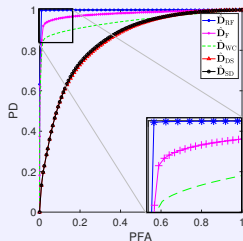
$$\Delta \mathbf{X}^{(k+1)} = \arg \min_{\Delta \mathbf{X}} \left\| \Lambda_{\text{HR}}^{-\frac{1}{2}} \left(\Delta \tilde{\mathbf{Y}}_{\text{HR}}^k - \mathbf{L} \Delta \mathbf{X} \right) \right\|_{\text{F}}^2 + \gamma \|\Delta \mathbf{X}\|_{2,1}$$

with

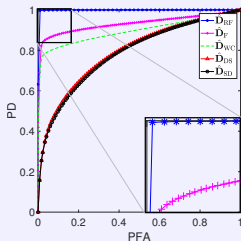
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Experiments on synthetic images

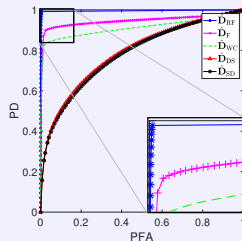
Detection performance



(a) Situation 1: HR-MS/LR-HS



(b) Situation 2: HR-PAN/LR-HS



(c) Situation 3: HR-PAN/LR-MS

Table: Situations 1 , 2 & 3: quantitative detection performance (AUC and distance).

		$\bar{D}_{RF}$	$\bar{D}_F$	$\bar{D}_{WC}$	$\bar{D}_{DS}$	$\bar{D}_{SD}$
Situation 1	AUC	0.997469	0.981039	0.941408	0.843685	0.847518
	Dist.	0.990299	0.951995	0.887789	0.766677	0.771277
Situation 2	AUC	0.997418	0.931047	0.89517	0.790859	0.785019
	Dist.	0.990299	0.883488	0.833783	0.718072	0.712771
Situation 3	AUC	0.994929	0.94522	0.911311	0.786255	0.779522
	Dist.	0.991699	0.915992	0.864686	0.713471	0.706871

Experiments on real images

Data description

Observed image at t_1 (04/15/2015):

- Local: Lake-Tahoe (CA) USA.
- Sensor: Landsat 8.
- Image size: 175×180 pixels.
- Spatial resolution: 30m per pixel.
- Spectral resolution: 3 spectral bands (MS) in RGB visible spectrum .

Observed image at t_2 (09/22/2015):

- Local: Lake-Tahoe (CA) USA.
- Sensor: Landsat 8.
- Image size: 350×360 pixels.
- Spatial resolution: 15m per pixel.
- Spectral resolution: PAN in RGB visible spectrum .

Preprocessing:

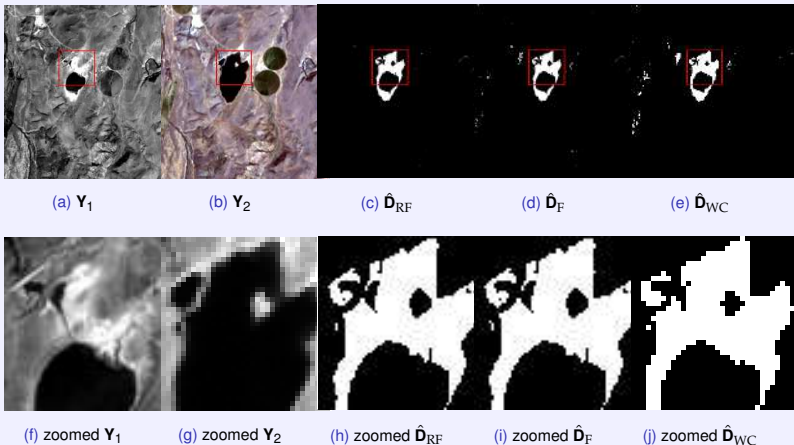
- Manual alignment.

Compared Methods:

- Robust Fusion approach ($\hat{\mathbf{D}}_{\text{RF}}$).
- Fusion approach ($\hat{\mathbf{D}}_{\text{F}}$) .
- Worst-case approach ($\hat{\mathbf{D}}_{\text{WC}}$).

Experiments on real images

Visual results



Scenario $\mathcal{S}_4$: (a) LR-MS observed image Y_{LR} , (b) HR-PAN observed image Y_{HR} , (e) change mask $\hat{D}_{WC}$ estimated by the WC approach, (d) change mask $\hat{D}_F$ estimated by the fusion approach and (c) change mask $\hat{D}_{RF}$ estimated by the proposed approach. From (f) to (h): zoomed versions of the regions delineated in red in (a)–(c).

Outline

Introduction

Multi-band optical image fusion

- Problem statement

- Fast fusion solving a Sylvester equation

- Experiments

Multi-band optical image change detection

- Fusion approach

- Robust Fusion approach

Conclusions

Fusion

- fusion of multi-band images formulated as a **linear inverse problem**
- spectral regularization: constraining the estimation in a lower-dimensional space
- spatial regularizations:
 - Gaussian prior
 - dictionary-based sparse prior
 - ...
- explicit solution under generalized Thinonov regularizations, which can be embedded into iterative algorithms
 - for more complex priors
 - when estimating jointly other parameters (noise variance, spectral response,...)
- assumes spectrally-invariant spatial blurs... not valid for astrophysical data
→ see [GOBD20] and [GOB⁺20]
- fusion as a convenient framework to address change detection problems

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- spectral regularization: constraining the estimation in a lower-dimensional space
- spatial regularizations:
 - Gaussian prior
 - dictionary-based sparse prior
 - ...
- explicit solution under generalized Thinonov regularizations, which can be embedded into iterative algorithms
 - for more complex priors
 - when estimating jointly other parameters (noise variance, spectral response,...)
- assumes spectrally-invariant spatial blurs... not valid for astrophysical data
→ see [GOBD20] and [GOB⁺20]
- fusion as a convenient framework to address change detection problems

Change detection

Fusion approach

- unsupervised CD between optical images of different spatial/spectral resolutions
- assumes prior knowledge on the forward model (degradations)
- steps tailored by the end-user
- provides 2 CD maps of different resolutions

Robust fusion approach

- unsupervised CD between optical images of different spatial/spectral resolutions
- assumes prior knowledge on the forward model (degradations)
- estimates high resolution latent images and CD images
- provides 1 CD map of high spatial and spectral resolutions

For images of different modalities (e.g., optical and SAR)

- no forward model available
- no physically interpretable latent space
- latent space identified by data-driven methods, e.g., dictionary learning [FDC⁺19]

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Multi-band optical imaging

From fusion to change detection

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