Multi-band optical imaging
From fusion to change detection

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Multi-band optical imaging

*Multi/hyper-spectral images*

- same scene observed at different wavelengths
Multi-band optical imaging

Multi/hyper-spectral images

- same scene observed at different wavelengths

Hyperspectral Cube
Introduction

Multi-band optical imaging

Multi/hyper-spectral images

- same scene observed at different wavelengths,
- pixel represented by a vector of tens/hundreds of measurements.
Multi-band optical imaging

**Multi/hyper-spectral images**

- same scene observed at different wavelengths,
- pixel represented by a vector of tens/hundreds of measurements.

**Hyperspectral Cube**
Introduction

Multi-band optical imaging

![Image of plants]
Introduction

Multi-band optical imaging

[Image of plants under 473 nm light]
Introduction

Multi-band optical imaging

547 nm
Introduction

Multi-band optical imaging

![Image of plants with the text "621 nm" indicating the wavelength]
Multi-band optical imaging
Introduction

Multi-band optical imaging

770 nm
**Multi-band optical imaging**
Spatial vs. spectral resolution trade-off

*Panchromatic images (PAN)*
- no spectral resolution (only 1 band),
- very high spatial resolution ($\sim 10$cm).

*Multispectral images (MS)*
- low spectral resolution ($\sim 10$ bands),
- high spatial resolution ($\sim 1$m).

*Hyperspectral images (HS)*
- high spectral resolution ($\sim 100$ bands),
- low spatial resolution ($\sim 10$m).
Multi-band optical imaging
Spatial vs. spectral resolution trade-off

Spot HS (20m)  Quickbird MS (4m)  Ikonos PAN (1m)
Introduction

Multi-band optical image fusion
  Problem statement
  Fast fusion solving a Sylvester equation
  Experiments

Multi-band optical image change detection
  Fusion approach
  Robust Fusion approach

Conclusions
Introduction

Multi-band optical image fusion
  Problem statement
    Fast fusion solving a Sylvester equation
      From maximum likelihood estimator...
      ... to maximum a posteriori estimators
  Experiments

Multi-band optical image change detection
  Fusion approach
  Robust Fusion approach

Conclusions
Multi-band optical image fusion

Multiple image fusion

**Pansharpening: PAN+MS fusion**
- incorporate the spatial details of the PAN image into the MS image
- huge literature
- main approaches rely on band substitution

**Hyperspectral pansharpening: PAN+HS fusion**
- incorporate the spatial details of the PAN image into the HS image
- more difficult due to the size of the HS image
- specific methods should be developed

**Multi-band image fusion: MS+HS fusion**
- incorporate the spatial details of the MS image into the HS image
- more difficult since the spatial details contained in a multi-band image
- specific methods should be developed
**Multiple image fusion**

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Problem statement

(a) Hyperspectral Image (size: $99 \times 46 \times 224$, res.: $20m \times 20m$) (b) Multispectral Image (size: $396 \times 184 \times 4$ res.: $5m \times 5m$) (c) Target (size: $396 \times 184 \times 224$ res.: $5m \times 5m$)

<table>
<thead>
<tr>
<th>Name</th>
<th>AVIRIS (HS)</th>
<th>SPOT-5 (MS)</th>
<th>Pleiades (MS)</th>
<th>WorldView-3 (MS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Res. (m)</td>
<td>20</td>
<td>10</td>
<td>2</td>
<td>1.24</td>
</tr>
<tr>
<td># bands</td>
<td>224</td>
<td>4</td>
<td>4</td>
<td>8</td>
</tr>
</tbody>
</table>

Table: Some existing remote sensors characteristics
Forward model for multi-band optical images

\[ T_D[\cdot] \quad T_N[\cdot] \]

\[ \begin{array}{c}
    X \quad \text{Spectral Degradation} \quad L \quad \text{Spatial Degradation} \quad R \quad \text{Y}
\end{array} \]

\[ Y = LXR + N \]

where

- **Y** observed multiband image of low spatial and/or spectral resolutions (row ↔ band, column ↔ pixel)
- **X** (unknown) latent image of high spatial and/or spectral resolutions (row ↔ band, column ↔ pixel)
- **L** spectral degradation matrix
- **R** spatial degradation matrix, e.g., decomposed as \( R = BS \) with
  - **B** spatial blur
  - **S** spatial subsampling
Complementary acquisitions: forward models

- \( \mathbf{X} \in \mathbb{R}^{m \times n} \): full resolution unknown image
Complementary acquisitions: forward models

- $X \in \mathbb{R}^{m \times n}$: full resolution unknown image
- $B \in \mathbb{R}^{n \times n}$: cyclic convolution operator acting on the bands
Complementary acquisitions: forward models

- \( \mathbf{X} \in \mathbb{R}^{m \times n} \): full resolution unknown image
- \( \mathbf{B} \in \mathbb{R}^{n \times n} \): cyclic convolution operator acting on the bands
- \( \mathbf{S} \in \mathbb{R}^{n \times m} \): downsampling operator
Complementary acquisitions: forward models

\[ Y_H \approx XBS \]

- \( X \in \mathbb{R}^{m\lambda \times n} \): full resolution unknown image
- \( Y_H \in \mathbb{R}^{m\lambda \times m} \): observed HS image
- \( B \in \mathbb{R}^{n \times n} \): cyclic convolution operator acting on the bands
- \( S \in \mathbb{R}^{n \times m} \): downsampling operator
Multi-band optical image fusion

Complementary acquisitions: forward models

\[ Y_H \approx XBS \]

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Spatial blur \( B \)
Complementary acquisitions: forward models

\[ Y_H \approx X_{BS} \]

- \( X \in \mathbb{R}^{m \times n} \): full resolution unknown image
- \( Y_H \in \mathbb{R}^{m \times m} \): observed HS image
- \( B \in \mathbb{R}^{n \times n} \): cyclic convolution operator acting on the bands
- \( S \in \mathbb{R}^{n \times m} \): downsampling operator
- \( R \in \mathbb{R}^{n \times m} \): spectral response of the MS sensor

![Spatial blur B and spectral response R](image-url)
Complementary acquisitions: forward models

\[ Y_H \approx XBS, \quad Y_M \approx RX \]

- \( X \in \mathbb{R}^{m \times n} \): full resolution unknown image
- \( Y_H \in \mathbb{R}^{m \times m} \): observed HS image
- \( Y_M \in \mathbb{R}^{n \times n} \): observed MS image
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- \( S \in \mathbb{R}^{n \times m} \): downsampling operator
- \( R \in \mathbb{R}^{n \times m} \): spectral response of the MS sensor
Complementary acquisitions: forward models

\[ Y_H = XBS + N_H, \quad Y_M = RX + N_M \]

- \( X \in \mathbb{R}^{m_\lambda \times n} \): full resolution unknown image
- \( Y_H \in \mathbb{R}^{m_\lambda \times m} \): observed HS image
- \( Y_M \in \mathbb{R}^{n_\lambda \times n} \): observed MS image
- \( B \in \mathbb{R}^{n \times n} \): cyclic convolution operator acting on the bands
- \( S \in \mathbb{R}^{n \times m} \): downsampling operator
- \( R \in \mathbb{R}^{n_\lambda \times m_\lambda} \): spectral response of the MS sensor
- \( N_H \in \mathbb{R}^{m_\lambda \times m} \) and \( N_M \in \mathbb{R}^{n_\lambda \times n} \): HS and MS noises
Noise statistics

Gaussian assumption

\[ \mathbf{N}_H | \mathbf{\Lambda}_H \sim \mathcal{MN}_{m \lambda, m}(\mathbf{0}_{m \lambda}, m, \mathbf{\Lambda}_H, \mathbf{I}_m) \]
\[ \mathbf{N}_M | \mathbf{\Lambda}_M \sim \mathcal{MN}_{n \lambda, n}(\mathbf{0}_{n \lambda}, n, \mathbf{\Lambda}_M, \mathbf{I}_n) \]

where

- \( \mathbf{\Lambda}_H = \text{diag} \left\{ s_{H,1}^2, \ldots, s_{H,m \lambda}^2 \right\} \) (hyperspectral noise variances)
- \( \mathbf{\Lambda}_M = \text{diag} \left\{ s_{M,1}^2, \ldots, s_{M,n \lambda}^2 \right\} \) (multispectral noise variances)

and the pdf of a matrix normal distribution is defined by

\[
p(\mathbf{Z} | \mathbf{\tilde{Z}}, \mathbf{\Sigma}_r, \mathbf{\Sigma}_c) \propto \exp \left( -\frac{1}{2} \text{tr} \left[ \mathbf{\Sigma}_c^{-1} (\mathbf{Z} - \mathbf{\tilde{Z}})^T \mathbf{\Sigma}_r^{-1} (\mathbf{Z} - \mathbf{\tilde{Z}}) \right] \right)
\]

\( \rightarrow \) band-dependent noise
\( \rightarrow \) pixel-independent noise
Likelihood of the observations

Given the forward model (characterized by both left- and right-operators)

\[
Y_H = XBS + N_H \\
Y_M = RX + N_M
\]

the two likelihood functions express as

\[
Y_H|X \sim \mathcal{MN}_{m, m}(XBS, \Lambda_H, I_m) \\
Y_M|X \sim \mathcal{MN}_{n, n}(RX, \Lambda_M, I_n)
\]

Joint likelihood
HS and MS images acquired by distinct sensors
→ independent HS and MS noises
→ independent observed images, cond. on \(X\)

\[
f(Y_H, Y_M|X) = f(Y_H|X) f(Y_M|X)
\]
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Joint likelihood

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\[ f(Y_H, Y_M|X) = f(Y_H|X) f(Y_M|X) \]
Multi-band image fusion as an estimation problem

*Maximum likelihood estimation*
Maximizing the two data-fitting terms writes

\[
\hat{X} \in \arg\min_X - \log f(Y_H|X) - \log f(Y_M|X)
\]

formulated as a weighted least-square regression

\[
\hat{X} \in \arg\min_X \|Y_M - RX\|_{\Lambda_M^{-1}}^2 + \|Y_H - XBS\|_{\Lambda_H^{-1}}^2
\]

*Main issues*
- (generally) large scale problem
- (generally) ill-posed (at least ill-conditioned) problem

*Regularization required...*
- (always) in the spectral domain
- (optional) in the spatial domain
Multi-band image fusion as an estimation problem

**Maximum likelihood estimation**
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**Main issues**
- (generally) large scale problem
- (generally) ill-posed (at least ill-conditioned) problem

**Regularization required**...
- (always) in the spectral domain
- (optional) in the spatial domain
Hyperspectral pixels live in a (much) lower-dimensional subspace...

Unknown image $X$ enforced to be decomposed as

$$X = HU$$

i.e., its pixels live in a lower-dimensional subspace ($\mathbb{R}^{\tilde{m}_\lambda}$ with $\tilde{m}_\lambda \ll m_\lambda$) spanned by the columns of $H \in \mathbb{R}^{m_\lambda \times \tilde{m}_\lambda}$ (estimated or known a priori).
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Given the spectral regularization, the optimization problem writes

\[
\hat{U} \in \arg\min_U \mathcal{J}(U)
\]

with

\[
\mathcal{J}(U) = \| \Lambda_H^{-1}(Y_H - HUBS) \|_F^2 + \| \Lambda_M^{-1}(Y_M - RHU) \|_F^2
\]

\[
\nabla \mathcal{J}(U) = 0 \iff \text{finding } U \text{ such that } C_1 U + UC_2 = C_3
\]

with

\[
C_1 = \left[H^H \Lambda_H^{-1} H\right]^{-1} \left[(RH)^H \Lambda_M^{-1} (RH)\right]
\]

\[
C_2 = \left[BS (BS)^H\right]
\]

\[
C_3 = \text{term depending on } Y_H \text{ and } Y_M \text{ (ind. on } U)\]
Solving a Sylvester matrix equation

\[ C_1 U + U C_2 = C_3 \]

Main issue
\[ C_2 = B S (B S)^H \] is not diagonalizable!

In the literature...
- **general resolution**: Bartels-Stewart algorithm (1972), with complexity of \( O(n^3) \) → impossible in practice
- **in the context of fusion**: iterative algos, e.g., gradient descent, ADMM... → time consuming

Our contribution
We showed that an explicit solution can be written and easily computed! (see [WDT15a, WDT+16])

Remark: result can be applied to (because generalizes) superresolution [ZWB+16]...
**Assumption 1**

The blurring matrix $B$ is a **block circulant matrix with circulant blocks (BCCB)**.

![Block Circulant Matrix](image)

**Assumption 2**

The decimation matrix $S$ corresponds to **downsampling** the original signal and its conjugate transpose $S^H$ **interpolates** the decimated signal **with zeros**.

**e.g.**

$$S = \begin{bmatrix}
1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\
\end{bmatrix}$$
Fast fUision based on a Sylvester Equation (FUSE)

Input: $Y_M, Y_H, \Lambda_M, \Lambda_H, R, B, S, H$

$D \leftarrow F^HBF$ and $D \leftarrow D^*D$  
/* Circulant matrix: $B = FDF^H */$

$C_1 \leftarrow \left( H^H \Lambda_H^{-1} H \right)^{-1} \left( (RH)^H \Lambda_L^{-1} RH \right)$  
/* Compute $C_1 */$

$(Q, \Lambda_C) \leftarrow \text{EigDec}(C_1)$  
/* Eigen-dec of $C_1$: $C_1 = Q\Lambda_CQ^{-1} */$

$\tilde{C}_3 \leftarrow Q^{-1} \left( H^H \Lambda_H^{-1} H \right)^{-1} \left( H^H \Lambda_H^{-1} Y_H (BS)^H + (RH)^H \Lambda_L^{-1} Y_M \right)BFP^{-1}$

for $l = 1$ to $m_\lambda$ do

\[ u_{l,1} = (\tilde{C}_3)_{l,1} \left( \frac{1}{d} \sum_{i=1}^{d} D_i + \lambda_C^l I_n \right)^{-1} \]

for $j = 2$ to $d$ do

\[ u_{l,j} = \frac{1}{\lambda_C^l} \left( (\tilde{C}_3)_{l,j} - \frac{1}{d} u_{l,1} D_j \right) \]

end

end

$\hat{U} = QUPD^{-1}F^H$

Output: $\hat{X} = H\hat{U}$
Incorporating (spatial) regularization

$$\hat{U} \in \arg\min_U \left\| \Lambda_H^{-1} (Y_H - HUBS) \right\|_F^2 + \left\| \Lambda_M^{-1} (Y_M - RHU) \right\|_F^2 + \mu \phi(U)$$

where

- $\phi(U) = \left\| \Gamma (U - \tilde{U}) \right\|_F^2$: (generalized) Thikonov regularizations
  + $\Gamma = I$ and $\tilde{U}$ = “crude estimate”: supervised naive Gaussian prior [WDT15b]  
    → closed-form solution
  + joint estimation of $\Lambda = \{ \Lambda_H, \Lambda_M \}$ and $\mu$: unsupervised naive Gaussian prior [WDT15c]  
    → closed-form solution embedded in BCD algorithm
Incorporating (spatial) regularization

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    \( \rightarrow \) closed-form solution embedded in BCD algorithm
Gaussian prior

Unsupervised naive Gaussian prior: closed-form solution embedded in BCD (FUSE-BCD)

\[ \text{Input: } Y_{H}, Y_{M}, \tilde{m}_{\lambda}, B, S, R, H \]
\[ \text{for } t = 1 \text{ to } T \text{ do} \]
\[ \quad \text{// Optimize w.r.t. to } U \]
\[ \quad U_t = \arg \min_U L(U, \Lambda_{t-1}, \mu_{t-1}) \]
\[ \quad \text{// Sylvester equation } \]
\[ \quad \text{// Optimize w.r.t. } \Lambda \]
\[ \quad \Lambda_t = \arg \min_{\Lambda} L(U_t, \Lambda, \mu_{t-1}) \]
\[ \quad \text{// Optimize w.r.t. } \mu \]
\[ \quad \mu_t = \arg \min_{\mu} L(U_t, \Lambda_t, \mu) \]
\[ \text{end} \]
\[ \hat{U} \leftarrow U_T \]
\[ \text{Output: } \hat{X} = H\hat{U} \]
Incorporating (spatial) regularization

\[ \hat{U} \in \text{argmin}_U \left\| \Lambda_H^{-1} (Y_H - \text{HUBS}) \right\|_F^2 + \left\| \Lambda_M^{-1} (Y_M - \text{RHU}) \right\|_F^2 + \mu \phi(U) \]

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- \( \phi(U) = \| U - DA \|_F^2 \): sparse representation based on dictionary learning [WBDT15]
  - \( D \): dictionary learnt beforehand
  - \( A \): code estimated (with sparse support learnt beforehand)
  → closed-form solution embedded in BCD algorithm
Incorporating (spatial) regularization

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Sparse representation

Sparse prior: closed-form solution embedded in BCD (FUSE-BCD)

**Input:** $Y_H, Y_M, \tilde{m}_\lambda, B, S, R, H, \text{SNR}_H, \text{SNR}_M, n_{\text{max}}$

// Rough estimation of $U$

Approximate $\tilde{U}$ using $Y_M$ and $Y_H$;

// Online dictionary learning

$\hat{D} \leftarrow \text{ODL}(\tilde{U})$;

// Sparse coding

$\hat{A} \leftarrow \text{OMP}(\hat{D}, \tilde{U}, n_{\text{max}})$;

// Computing support

$\hat{\Omega} \leftarrow \hat{A} \neq 0$;

// Start alternate optimization

for $t = 1$ to $T$ do

// Optimize w.r.t. to $U$

$U_t = \arg\min_U L(U, A_{t-1})$; /* Sylvester equation */

// Optimize w.r.t. to $A$

$A_t = \arg\min_U L(U_t, A)$; /* solved with LS */

end

Output: $\hat{X} = H\hat{U}$
Incorporating (spatial) regularization

\[ \hat{U} \in \text{argmin}_U \left\| \Lambda_H^{-1} (Y_H \ominus HUBS) \right\|_F^2 + \left\| \Lambda_M^{-1} (Y_M \ominus RHU) \right\|_F^2 + \mu \phi(U) \]

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    - \( \rightarrow \) closed-form solution embedded in BCD algorithm

- \( \phi(U) = \sum_{j=1}^{m\lambda} \text{TV} \left[ U_{j,:} \right] \): band-wise total variation [SoBAC15]
  - \( \rightarrow \) closed-form solution embedded in ADMM algorithm
Multi-band optical image fusion

Fast fusion solving a Sylvester equation

From ML to MAP estimators

Incorporating (spatial) regularization

\[
\hat{U} \in \arg\min_U \left\| \Lambda_H^{-1} (Y_H - \text{HUBS}) \right\|_F^2 + \left\| \Lambda_M^{-1} (Y_M - \text{RHU}) \right\|_F^2 + \mu \phi(U)
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- \( \phi(U) = \sum_{j=1}^{m} \lambda \text{TV} [U_{j,:}] \): band-wise total variation [SoBAC15]
  \( \rightarrow \) closed-form solution embedded in ADMM algorithm
Non-Gaussian prior, such as TV

$$\arg\min_u \left\{ \frac{1}{2} \left\| \Lambda_H^{-\frac{1}{2}} (Y_H - \text{HUBS}) \right\|_F^2 + \frac{1}{2} \left\| \Lambda_M^{-\frac{1}{2}} (Y_M - \text{RHU}) \right\|_F^2 + \lambda \text{TV} (U) \right\}.$$  

HS data term  
MS data term  
regularizer

can be equivalently solved as:

$$\arg\min_{u,v} \left\{ \frac{1}{2} \left\| \Lambda_H^{-\frac{1}{2}} (Y_H - \text{HUBS}) \right\|_F^2 + \frac{1}{2} \left\| \Lambda_M^{-\frac{1}{2}} (Y_M - \text{RHU}) \right\|_F^2 + \lambda \text{TV} (V) \text{ s.t. } U = V \right\}.$$  

- ADMM algorithm: alternate minimization (FUSE-ADMM)
  - closed-form solution of the Sylvester equation
  - proximal mapping
Illustrative results
PAN + HS fusion / Gaussian prior

(left to right) HS image, PAN image, ground truth, ADMM, proposed method.
Performance and computational times
HS + MS fusion / various regularizations

Table: RSNR (in dB), UIQI, SAM (in degree), ERGAS, DD (in $10^{-3}$) and time (in second).

<table>
<thead>
<tr>
<th>Regularization</th>
<th>Methods</th>
<th>RSNR</th>
<th>UIQI</th>
<th>SAM</th>
<th>ERGAS</th>
<th>DD</th>
<th>Time</th>
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<tbody>
<tr>
<td>supervised</td>
<td>ADMM</td>
<td>29.321</td>
<td>0.9906</td>
<td>1.555</td>
<td>0.888</td>
<td>7.115</td>
<td>126.83</td>
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<tr>
<td>naive Gaussian</td>
<td>FUSE</td>
<td>29.372</td>
<td>0.9908</td>
<td>1.551</td>
<td>0.879</td>
<td>7.092</td>
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<td>unsupervised</td>
<td>ADMM-BCD</td>
<td>29.084</td>
<td>0.9902</td>
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<td>0.913</td>
<td>7.341</td>
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<td>naive Gaussian</td>
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<td>29.077</td>
<td>0.9902</td>
<td>1.623</td>
<td>0.913</td>
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<tr>
<td>sparse representation</td>
<td>ADMM-BCD</td>
<td>29.582</td>
<td>0.9911</td>
<td>1.423</td>
<td>0.872</td>
<td>6.678</td>
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<td></td>
<td>FUSE-BCD</td>
<td>29.688</td>
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<td>TV</td>
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<td></td>
<td>FUSE-ADMM</td>
<td>29.631</td>
<td>0.9915</td>
<td>1.477</td>
<td>0.845</td>
<td>6.788</td>
<td>90.99</td>
</tr>
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</table>

- The computational time is decreased significantly!
Comparison with state-of-the-art methods
PAN + HS fusion

Table: Characteristics of the three datasets [LBDB+15]

<table>
<thead>
<tr>
<th>dataset</th>
<th>dimensions</th>
<th>spatial res</th>
<th>N</th>
<th>instrument</th>
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<tbody>
<tr>
<td>Moffett</td>
<td>PAN 185 × 395</td>
<td>20m</td>
<td>224</td>
<td>AVIRIS</td>
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<td></td>
<td>HS 37 × 79</td>
<td>100m</td>
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<td></td>
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<tr>
<td>Camargue</td>
<td>PAN 500 × 500</td>
<td>4m</td>
<td>125</td>
<td>HyMap</td>
</tr>
<tr>
<td></td>
<td>HS 100 × 100</td>
<td>20m</td>
<td></td>
<td></td>
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<tr>
<td>Garons</td>
<td>PAN 400 × 400</td>
<td>4m</td>
<td>125</td>
<td>HyMap</td>
</tr>
<tr>
<td></td>
<td>HS 80 × 80</td>
<td>20m</td>
<td></td>
<td></td>
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</tbody>
</table>
## Comparison with state-of-the-art methods

**PAN + HS fusion**

<table>
<thead>
<tr>
<th>method</th>
<th>CC</th>
<th>SAM</th>
<th>RMSE</th>
<th>ERGAS</th>
<th>Time (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SFIM</td>
<td>0.92955</td>
<td>9.5271</td>
<td>365.2577</td>
<td>6.5429</td>
<td>1.26</td>
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<tr>
<td>MTF-GLP</td>
<td>0.93919</td>
<td>9.4599</td>
<td>352.1290</td>
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<td>PCA</td>
<td>0.89580</td>
<td>14.6132</td>
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<td>GFPCA</td>
<td>0.91614</td>
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<td>404.2979</td>
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<td>CNMF</td>
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<td>Supervised Gaussian</td>
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<td>HySure</td>
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</table>
Comparison with state-of-the-art methods
PAN + HS fusion

Table: Quality measures for the Camargue dataset

<table>
<thead>
<tr>
<th>method</th>
<th>CC</th>
<th>SAM</th>
<th>RMSE</th>
<th>ERGAS</th>
<th>Time (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SFIM</td>
<td>0.91886</td>
<td>4.2895</td>
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<td>GSA</td>
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<td>GFPCA</td>
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## Comparison with state-of-the-art methods

### PAN + HS fusion

**Table:** Quality measures for the Garons dataset

<table>
<thead>
<tr>
<th>method</th>
<th>CC</th>
<th>SAM</th>
<th>RMSE</th>
<th>ERGAS</th>
<th>Time (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SFIM</td>
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<td>MTF-GLP-HPM</td>
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<td>23.98</td>
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<td>Supervised Gaussian</td>
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<td>3.07</td>
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<td>6.0224</td>
<td>778.1051</td>
<td>4.0454</td>
<td>177.60</td>
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Outline

Introduction

Multi-band optical image fusion
  Problem statement
  Fast fusion solving a Sylvester equation
  Experiments

Multi-band optical image change detection
  Fusion approach
    Problem statement
    Resolution pipeline
    Experiments
  Robust Fusion approach
    Problem statement
    Algorithm
    Experimental results

Conclusions
Multi-band optical image change detection

Change Detection (CD)

Input

- Two or more multitemporal images.
- Same geographical spot (scene).

Output

- Change map.

Source: RafaelRabellodeBarros
Multi-band optical image change detection

CD in remote sensing context

Applications:
- Land-use and land-cover analysis.
- Urban monitoring.
- Environmental surveillance.
- Defense and security.

Taxonomy of methods:
- According to supervision.
  - Supervised.
  - Unsupervised.
- According to modality.
  - Same modality.
  - Multimodality.
Supervised vs. Unsupervised

**Supervised**
- Require ground information.
- More appropriate to multimodal images.
- Higher complexity of methods.
- Good overall performance.
- Depend on training set.
- Less appealing for real applications.

**Unsupervised**
- Does not require any ground information.
- Generally applied to same modalities.
- Lower complexity of methods.
- Lower overall performance.
- Generally require preprocessing steps.
- Automatic behaviour.
Favorable scenario

- Same modality.
- Identical resolutions.

Comparison of homologous pixels!

Same modality CD!
Favorable scenario

Landsat 8 04/15/2015 (PAN - 15m)

Landsat 8 09/22/2015 (PAN - 15m)

- Same modality.
- Identical resolutions.

Comparison of homologous pixels!

Same modality CD!
Unfavorable scenario

Emergence situation:
- Natural disaster.
- Punctual missions.
- Defense and security.

Need:
- Multimodal CD.

Landsat 8 04/15/2015 (MS - 30m)  
Landsat 8 09/22/2015 (PAN - 15m)
State-of-the-art

Principle:

- Identical resolutions obtained through independent and individual transformation over the considered images. [KCS+13]
- Multimodality CD achieved by supervised or semi-supervised methods [PCP+15].

Methods:

- Worst-case (WC): Degradation of both observed images.
- Degradation-Superresolution (DS): Spectral degradation followed by spatial Superresolution of LR-HS observed image.
- Superresolution-Degradation (SD): Spatial Superresolution followed by spectral degradation of LR-HS observed image.
- Coupled dictionary learning [GZSL16].

Problems:

- No joint processing.
- Loss of information (DS and SD).
- Loss of resolution (WC).
Adopted strategy: leveraging on fusion

General principle:
- Consider two images of same region acquired at same time (no change).
- Fused image evidences information contained in the pair of input images.
- Able to deal with different resolutions (e.g. pansharpening).

Today:
- Fusion-based approach [FDWC18]
- Robust-fusion based approach [FDWC17]

Remarks:
- Easier to manipulate optical images due to the noise statistics.
- 85% of the total earth observation satellites are optical [oCS17].
Multi-band optical image change detection

Fusion approach

Forward model for multi-band optical images

\[ Y = LXR + N \]

where

- \( Y \) observed multiband image of low spatial and/or spectral resolutions (row \( \leftrightarrow \) band, column \( \leftrightarrow \) pixel)
- \( X \) (unknown) latent image of high spatial and/or spectral resolutions (row \( \leftrightarrow \) band, column \( \leftrightarrow \) pixel)
- \( L \) spectral degradation matrix
- \( R \) spatial degradation matrix, e.g., decomposed as \( R = BS \) with
  - \( B \) spatial blur
  - \( S \) spatial subsampling
# Applicative scenarios

<table>
<thead>
<tr>
<th>Forward model #1</th>
<th>Forward model #2</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Spectral degradation</strong></td>
<td><strong>Spatial degradation</strong></td>
<td><strong>Spectral degradation</strong></td>
</tr>
<tr>
<td>$S_1$</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>$S_2$</td>
<td>$L_1$</td>
<td>—</td>
</tr>
<tr>
<td>$S_3$</td>
<td>—</td>
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<td>—</td>
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<td>$S_8$</td>
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</tr>
<tr>
<td>$S_9$</td>
<td>$L_1$</td>
<td>$R_1$</td>
</tr>
<tr>
<td>$S_{10}$</td>
<td>$L_1$</td>
<td>$R_1$</td>
</tr>
</tbody>
</table>

**Table:** Overview of the spectral and spatial degradations w.r.t. experimental scenarios. The symbol — stands for “no degradation” [FDC20].
### Applicative scenarios

<table>
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<tr>
<th>Forward model $#1$</th>
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<th>Comments</th>
</tr>
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<td>Spectral degradation</td>
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<td>Spectral degradation</td>
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<tr>
<td>$S_1$</td>
<td>$-$</td>
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<td>$S_4$</td>
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<tr>
<td>$S_{10}$</td>
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<td>$R_1$</td>
</tr>
</tbody>
</table>

**Table:** Overview of the spectral and spatial degradations w.r.t. experimental scenarios. The symbol $-$ stands for “no degradation” [FDC20].
Multi-band optical image change detection

Fusion approach

Scenario $S_4$

Landsat 8 04/15/2015 (MS - 30m)

\[ Y_{LR} = XR + N_{LR} \]

Landsat 8 09/22/2015 (PAN - 15m)

\[ Y_{HR} = LX + N_{HR} \]

- Time ordering independent: $t_1 \neq t_2$. 
Problem statement

Joint observation model

\[
\begin{align*}
Y_{LR} &= XR + N_{LR} \\
Y_{HR} &= LX + N_{HR}
\end{align*}
\]

Fusion process

\[
\hat{X} \leftarrow \text{FUSION} \left( Y_{LR}, Y_{HR} \right)
\]

“Predicted” pseudo-observed images

\[
\begin{align*}
\hat{Y}_{LR} &\triangleq \hat{X}R \\
\hat{Y}_{HR} &\triangleq L\hat{X}
\end{align*}
\]

Fusion properties [LdBD+ 15, WRM97]

- **Synthesis**: fused image \( \approx \) image obtained by the sensor of the target resolution.
- **Consistency**: reversibility of the fusion process.

Consistency-based CD hypothesis testing

\[
\begin{align*}
\mathcal{H}_0 : \left\{ \begin{array}{c}
Y_{LR} = \hat{Y}_{LR} \\
Y_{HR} = \hat{Y}_{HR}
\end{array} \right\} & \text{(no change)} \\
\mathcal{H}_1 : \left\{ \begin{array}{c}
Y_{LR} \neq \hat{Y}_{LR} \\
Y_{HR} \neq \hat{Y}_{HR}
\end{array} \right\} & \text{(change)}
\end{align*}
\]
Multi-band optical image change detection

Fusion approach

Problem statement

Joint observation model

\[
\begin{align*}
Y_{LR} &= X_R + N_{LR} \\
Y_{HR} &= L_X + N_{HR}
\end{align*}
\]

Fusion process

\[\hat{X} \leftarrow \text{FUSION}(Y_{LR}, Y_{HR})\]

“Predicted” pseudo-observed images

\[
\begin{align*}
\hat{Y}_{LR} &\triangleq \hat{X}_R \\
\hat{Y}_{HR} &\triangleq L\hat{X}
\end{align*}
\]

Fusion properties [LdB\textsuperscript{+}15, WRM97]

- Synthesis: fused image \(\approx\) image obtained by the sensor of the target resolution.
- Consistency: reversibility of the fusion process.

Consistency-based CD hypothesis testing

\[
\begin{align*}
H_0 : \quad &\begin{cases} 
Y_{LR} = \hat{Y}_{LR} \\
Y_{HR} = \hat{Y}_{HR}
\end{cases} & \text{(no change)} \\
H_1 : \quad &\begin{cases} 
Y_{LR} \neq \hat{Y}_{LR} \\
Y_{HR} \neq \hat{Y}_{HR}
\end{cases} & \text{(change)}
\end{align*}
\]
Problem statement

Joint observation model

\[ Y_{LR} = X_R + N_{LR} \]
\[ Y_{HR} = L_X + N_{HR} \]

Fusion process

\[ \hat{X} \leftarrow \text{FUSION} (Y_{LR}, Y_{HR}) \]

“Predicted” pseudo-observed images

\[ \hat{Y}_{LR} \triangleq \hat{X}_R \]
\[ \hat{Y}_{HR} \triangleq \hat{L}_X \]

Fusion properties [LdBD\textsuperscript{+} 15, WRM97]

- Synthesis: fused image \( \approx \) image obtained by the sensor of the target resolution.
- Consistency: reversibility of the fusion process.

Consistency-based CD hypothesis testing

\[ H_0 : \left\{ \begin{array}{c}
Y_{LR} = \hat{Y}_{LR} \\
Y_{HR} = \hat{Y}_{HR}
\end{array} \right. \quad \text{(no change)} \]

\[ H_1 : \left\{ \begin{array}{c}
Y_{LR} \neq \hat{Y}_{LR} \\
Y_{HR} \neq \hat{Y}_{HR}
\end{array} \right. \quad \text{(change)} \]
Problem statement

**Joint observation model**

\[
Y_{LR} = XR + N_{LR} \\
Y_{HR} = LX + N_{HR}
\]

**Fusion process**

\[\hat{X} \leftrightarrow \text{FUSION} (Y_{LR}, Y_{HR})\]

“Predicted” pseudo-observed images

\[
\hat{Y}_{LR} \triangleq \hat{X}R \\
\hat{Y}_{HR} \triangleq L\hat{X}
\]

**Fusion properties** [LdB\(^{+}\) 15, WRM97]

- **Synthesis**: fused image \(\approx\) image obtained by the sensor of the target resolution.
- **Consistency**: reversibility of the fusion process.

**Consistency-based CD hypothesis testing**

\[\mathcal{H}_0 : \left\{ \begin{array}{l}
Y_{LR} = \hat{Y}_{LR} \\
Y_{HR} = \hat{Y}_{HR}
\end{array} \right\} \text{ (no change)}\]

\[\mathcal{H}_1 : \left\{ \begin{array}{l}
Y_{LR} \neq \hat{Y}_{LR} \\
Y_{HR} \neq \hat{Y}_{HR}
\end{array} \right\} \text{ (change)}\]
Multi-band optical image change detection

3-steps procedure [FDWC18]

1. **fusion**: estimating $\hat{X}$ from $Y_{LR}$ and $Y_{HR}$
   - Tailored by the end user [FYDC18]
   - e.g., Fast fusion based on solving a SE

2. **prediction**: reconstructing $\hat{Y}_{LR}$ and $\hat{Y}_{HR}$ from $\hat{X}$
   $$
   \hat{Y}_{LR} = \hat{X}R
   $$
   $$
   \hat{Y}_{HR} = L\hat{X}.
   $$

3. **decision**: deriving change maps $D_{LR}$ and $D_{HR}$ from, resp.,
   $$
   \gamma_{LR} = \{Y_{LR}, \hat{Y}_{LR}\},
   $$
   $$
   \gamma_{HR} = \{Y_{HR}, \hat{Y}_{HR}\}.
   $$
   - Tailored by the end user.
   - e.g., Change Vector Analysis (CVA) [BB07]
Experiments on synthetic images
Detection performance

(a) Situation 1: HR-MS/LR-HS
(b) Situation 2: HR-PAN/LR-HS
(c) Situation 3: HR-PAN/LR-MS

Table: Situations 1, 2 & 3: quantitative detection performance (AUC and distance).

<table>
<thead>
<tr>
<th></th>
<th>$D_{HR}$</th>
<th>$D_{LR}$</th>
<th>$D_{aLR}$</th>
<th>$D_{WC}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Situation 1</td>
<td>AUC 0.981039</td>
<td>Dist. 0.951995</td>
<td>0.867478 0.789379</td>
<td>0.992242 0.979298</td>
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<td>Situation 2</td>
<td>AUC 0.931047</td>
<td>Dist. 0.883488</td>
<td>0.819679 0.737274</td>
<td>0.977362 0.952995</td>
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<tr>
<td>Situation 3</td>
<td>AUC 0.94522</td>
<td>Dist. 0.915992</td>
<td>0.711167 0.647865</td>
<td>0.984833 0.972997</td>
</tr>
</tbody>
</table>
Experiments on real images
Data description

Observed image at $t_1$ (04/15/2015):
- Local: Lake-Tahoe (CA) USA.
- Sensor: Landsat 8.
- Image size: 175 $\times$ 180 pixels.
- Spatial resolution: 30m per pixel.
- Spectral resolution: 3 spectral bands (MS) in RBG visible spectrum.

Preprocessing:
- Manual alignment.

Observed image at $t_2$ (09/22/2015):
- Local: Lake-Tahoe (CA) USA.
- Sensor: Landsat 8.
- Image size: 350 $\times$ 360 pixels.
- Spatial resolution: 15m per pixel.
- Spectral resolution: PAN in RBG visible spectrum.

Compared Methods:
- Fusion approach ($\hat{D}_{HR}, \hat{D}_{aLR}$).
- Worst-case approach ($\hat{D}_{WC}$).
Real scenario (LR-MS and HR-PAN): (a) LR-MS observed image $Y_{LR}$, (b) HR-PAN observed image $Y_{HR}$, (c) change mask $\mathbf{D}_{HR}$, (d) change mask $\mathbf{D}_{aLR}$, (e) change mask $\mathbf{D}_{WC}$ estimated by the worst-case approach. From (f) to (j): zoomed versions of the regions delineated in red in (a)–(e).
**Problem statement**

*Joint observation model: from fusion...*

\[
Y_{LR} = X_R + N_{LR} \\
Y_{HR} = LX + N_{HR}
\]

with

- \(X_1\): latent image at \(t_1\)
- \(X_2\): latent image at \(t_2\)

or, equivalently, \(X_2 = X_1 + \Delta X\) with

- \(\Delta X\): change image

\[
\Delta X = [\Delta x_1, \ldots, \Delta x_n] \quad \text{and} \quad \Delta x_i = [\Delta x_{1,i}, \ldots, \Delta x_{m,i}]^T
\]

*CD hypothesis testing*

Decision rule for the \(i\)th pixel \((i = 1, \ldots, n)\)

\[
\mathcal{H}_0 : \|\Delta x_i\|_2 < \tau \quad \text{(no change)}
\]

\[
\mathcal{H}_1 : \|\Delta x_i\|_2 \geq \tau \quad \text{(change)}
\]
Problem statement

Joint observation model: from fusion... to robust fusion

\[ Y_{LR} = X_1 R + N_{LR} \]
\[ Y_{HR} = LX_2 + N_{HR} \]

with

- \( X_1 \): latent image at \( t_1 \)
- \( X_2 \): latent image at \( t_2 \)

or, equivalently, \( X_2 = X_1 + \Delta X \) with

- \( \Delta X \): change image

\[ \Delta X = [\Delta x_1, \ldots, \Delta x_n] \text{ and } \Delta x_i = [\Delta x_{1,i}, \ldots, \Delta x_{m,i}]^T \]

CD hypothesis testing

Decision rule for the \( i \)th pixel \( (i = 1, \ldots, n) \)

\[ H_0 : \|\Delta x_i\|_2 < \tau \quad (\text{no change}) \]
\[ H_1 : \|\Delta x_i\|_2 \geq \tau \quad (\text{change}) \]
**Problem statement**

*Joint observation model: from fusion... to robust fusion*

\[
Y_{LR} = X_1 R + N_{LR} \\
Y_{HR} = L X_2 + N_{HR}
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- \(\Delta X\): change image

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\Delta X = [\Delta x_1, \ldots, \Delta x_n] \quad \text{and} \quad \Delta x_i = \left[\Delta x_{1,i}, \ldots, \Delta x_{m,i}\right]^T
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**Problem statement**

*Joint observation model: from fusion... to robust fusion*

\[
\begin{align*}
Y_{LR} &= X_1 R + N_{LR} \\
Y_{HR} &= L X_2 + N_{HR}
\end{align*}
\]

with
- \(X_1\): latent image at \(t_1\)
- \(X_2\): latent image at \(t_2\)

or, equivalently, \(X_2 = X_1 + \Delta X\) with
- \(\Delta X\): change image

\[
\Delta X = [\Delta x_1, \ldots, \Delta x_n] \quad \text{and} \quad \Delta x_i = \begin{bmatrix} \Delta x_{1,i}, \ldots, \Delta x_{m,i} \end{bmatrix}^T
\]

**CD hypothesis testing**

Decision rule for the \(i\)th pixel \((i = 1, \ldots, n)\)

\[
\begin{align*}
\mathcal{H}_0 : \|\Delta x_i\|_2 < \tau \quad &\text{(no change)} \\
\mathcal{H}_1 : \|\Delta x_i\|_2 \geq \tau \quad &\text{(change)}
\end{align*}
\]
Multi-band optical image change detection  
Robust Fusion approach

Optimization problem

Likelihood of the observations

\[
\begin{align*}
Y_{LR \mid X_1} & \sim \mathcal{M}\mathcal{N}_{\lambda_1 \cdot m_1}(X_1 R, \Lambda_{LR}, I_{m_1}) \\
Y_{HR \mid X_2} & \sim \mathcal{M}\mathcal{N}_{\lambda_2 \cdot n}(L X_2, \Lambda_{HR}, I_{n})
\end{align*}
\]

Maximum a posteriori (MAP) estimator

\[
\left\{ \hat{X}_{1,\text{MAP}}, \Delta \hat{X}_{\text{MAP}} \right\} \in \arg \min_{X_1, \Delta X} \left[ \Lambda_{LR}^{-\frac{1}{2}} \right] \left( Y_{LR} - X_1 R \right) \left\| F \right. \\
+ \left\| \Lambda_{HR}^{-\frac{1}{2}} \left( Y_{HR} - L (X_1 + \Delta X) \right) \right\| F_2 \\
+ \mu \phi_1 (X_1) + \gamma \phi_2 (\Delta X)
\]

Key ingredient: $\phi_2$

Spatial sparsity of the changes through a group-lasso regularization

\[
\phi_2 (\Delta X) = \| \Delta X \|_{2,1} = \sum_{i=1}^{n} \| \Delta x_i \|_2 = \| e \|_1
\]

promoting a sparse change image energy vector $e = [\| \Delta x_1 \|_2, \ldots, \| \Delta x_n \|_2]$.  

Nicolas Dobigeon  
ORASIS 2021, Lac de St-Ferréol
Optimization problem

Likelihood of the observations

\[ Y_{LR} | X_1 \sim \mathcal{M}\mathcal{N}_{n, \lambda_1} (X_1 R, \Lambda_{LR}, I_m) \]
\[ Y_{HR} | X_2 \sim \mathcal{M}\mathcal{N}_{m, \lambda_2} (LX_2, \Lambda_{HR}, I_n) \]

Maximum a posteriori (MAP) estimator

\[
\left\{ \hat{X}_{1, MAP}, \Delta \hat{X}_{MAP} \right\} \in \arg \min_{X_1, \Delta X} \left\| \Lambda_{LR}^{-\frac{1}{2}} (Y_{LR} - X_1 R) \right\|_F^2 \\
+ \left\| \Lambda_{HR}^{-\frac{1}{2}} (Y_{HR} - L (X_1 + \Delta X)) \right\|_F^2 \\
+ \phi_1 (X_1) + \gamma \phi_2 (\Delta X)
\]

Key ingredient: \( \phi_2 \)

Spatial sparsity of the changes through a group-lasso regularization

\[
\phi_2 (\Delta X) = \| \Delta X \|_{2,1}
\]
\[
= \sum_{i=1}^{n} \| \Delta x_i \|_2 = \| e \|_1
\]

promoting a sparse change image energy vector \( e = \left[ \| \Delta x_1 \|_2, \ldots, \| \Delta x_n \|_2 \right] \).
Solution

Robust fusion: alternate minimization algorithm [FDWC17, FDC20]

| Data: $Y_{HR}, Y_{LR}, L, R$  
| Input: $\Delta X^0$  
| for $k = 1, \ldots, K$ do  
| // Fusion step  
| $X_1^{(k+1)} = \arg \min_{X_1} \mathcal{J}(X_1, \Delta X^{(k)})$  
| // Correction step  
| $\Delta X^{(k+1)} = \arg \min_{\Delta X} \mathcal{J}(X_1^{(k+1)}, \Delta X)$  
| end  
| $\hat{X}_{1,\text{MAP}} \triangleq X_1^{(K+1)}$ and $\Delta \hat{X}_{\text{MAP}} \triangleq \Delta \hat{X}^{(K+1)}$  

Output: $\hat{X}_{1,\text{MAP}}, \Delta \hat{X}_{\text{MAP}}$

**Characteristics**

- Iterative minimization,
- Problem split into 2 simple sub-problems,
- Convergence guarantee.
Multi-band optical image change detection

Robust Fusion approach

**Optimization w.r.t. \( X_1 \)**

(fixing \( \Delta X = \Delta X^{(k)} \))

\[
\min_{X_1, \Delta X} \left\| \Lambda_{LR}^{-\frac{1}{2}} (Y_{LR} - X_1 R) \right\|^2_F + \left\| \Lambda_{HR}^{-\frac{1}{2}} (Y_{HR} - L (X_1 + \Delta X)) \right\|^2_F + \mu \| X_1 \|^2_F + \gamma \| \Delta X \|_{2,1}
\]

**Optimization problem**

\[
X_1^{(k+1)} = \arg \min_{X_1} \left\| \Lambda_{LR}^{-\frac{1}{2}} (Y_{LR} - X_1 R) \right\|^2_F + \left\| \Lambda_{HR}^{-\frac{1}{2}} (Y_{HR} - L (X_1 + \Delta X^{(k)})) \right\|^2_F + \mu \| X_1 \|^2_F
\]

rewritten as

\[
X_1^{(k+1)} = \arg \min_{X_1} \left\| \Lambda_{LR}^{-\frac{1}{2}} (Y_{LR} - X_1 R) \right\|^2_F + \left\| \Lambda_{HR}^{-\frac{1}{2}} (\tilde{Y}_{HR}^{(k)} - L X_1) \right\|^2_F + \mu \| X_1 \|^2_F
\]

with

- \( \tilde{Y}_{HR}^{(k)} = Y_{HR} - L \Delta X^{(k)} \): pseudo-observed image at \( t_1 \).
- Equivalent to an **image fusion** problem (single latent image estimation \( X_1 \)).
- Two quadratic data-fitting terms + Thikonov regularization
  - \( \to \) fast and explicit solution based on solving a **Sylvester equation**.
Multi-band optical image change detection

Robust Fusion approach

**Optimization w.r.t. $X_1$**

(fixing $\Delta X = \Delta X^{(k)}$)

$$\min_{X_1, \Delta X} \left\| \Lambda_{LR}^{-\frac{1}{2}} (Y_{LR} - X_1 R) \right\|_F^2 + \left\| \Lambda_{HR}^{-\frac{1}{2}} (Y_{HR} - L (X_1 + \Delta X)) \right\|_F^2 + \mu \| X_1 \|_F^2 + \gamma \| \Delta X \|_{2,1}$$

**Optimization problem**

$$X_1^{(k+1)} = \arg \min_{X_1} \left\| \Lambda_{LR}^{-\frac{1}{2}} (Y_{LR} - X_1 R) \right\|_F^2 + \left\| \Lambda_{HR}^{-\frac{1}{2}} (Y_{HR} - L (X_1 + \Delta X^{(k)})) \right\|_F^2 + \mu \| X_1 \|_F^2$$

rewritten as

$$X_1^{(k+1)} = \arg \min_{X_1} \left\| \Lambda_{LR}^{-\frac{1}{2}} (Y_{LR} - X_1 R) \right\|_F^2 + \left\| \Lambda_{HR}^{-\frac{1}{2}} (\bar{Y}_{HR}^{(k)} - LX_1) \right\|_F^2 + \mu \| X_1 \|_F^2$$

with

- $\bar{Y}_{HR}^{(k)} = Y_{HR} - L \Delta X^{(k)}$: pseudo-observed image at $t_1$.
- Equivalent to an image fusion problem (single latent image estimation $X_1$).
- Two quadratic data-fitting terms + Thikonov regularization
  $\rightarrow$ fast and explicit solution based on solving a Sylvester equation.
Optimization w.r.t. \( X_1 \)

\[
\min_{X_1, \Delta X} \left\| \Lambda_{LR}^{-\frac{1}{2}} (Y_{LR} - X_1 R) \right\|_F^2 + \left\| \Lambda_{HR}^{-\frac{1}{2}} (Y_{HR} - L (X_1 + \Delta X)) \right\|_F^2 + \mu \|X_1\|_F^2 + \gamma \|\Delta X\|_{2,1}
\]

Optimization problem

\[
X_{1}^{(k+1)} = \arg\min_{X_1} \left\| \Lambda_{LR}^{-\frac{1}{2}} (Y_{LR} - X_1 R) \right\|_F^2 + \left\| \Lambda_{HR}^{-\frac{1}{2}} (Y_{HR} - L (X_1 + \Delta X^{(k)})) \right\|_F^2 + \mu \|X_1\|_F^2
\]

rewritten as

\[
X_{1}^{(k+1)} = \arg\min_{X_1} \left\| \Lambda_{LR}^{-\frac{1}{2}} (Y_{LR} - X_1 R) \right\|_F^2 + \left\| \Lambda_{HR}^{-\frac{1}{2}} (\widetilde{Y}_HR^{(k)} - LX_1) \right\|_F^2 + \mu \|X_1\|_F^2
\]

with

- \( \widetilde{Y}_HR^{(k)} = Y_{HR} - L\Delta X^{(k)} \): pseudo-observed image at \( t_1 \).
- Equivalent to an image fusion problem (single latent image estimation \( X_1 \)).
- Two quadratic data-fitting terms + Thikonov regularization
  \( \rightarrow \) fast and explicit solution based on solving a Sylvester equation.
Multi-band optical image change detection

Robust Fusion approach

Optimization w.r.t. $\Delta X$

(fixing $X_1 = X_1^{(k)}$)

\[
\min_{X_1, \Delta X} \left\| \Lambda_{LR}^{-\frac{1}{2}} (Y_{LR} - X_1 R) \right\|_F^2 + \left\| \Lambda_{HR}^{-\frac{1}{2}} (Y_{HR} - L (X_1 + \Delta X)) \right\|_F^2 + \mu \|X_1\|_F^2 + \gamma \|\Delta X\|_{2,1}
\]

Optimization problem

\[
\Delta X^{(k+1)} = \arg \min_{\Delta X} \left\| \Lambda_{HR}^{-\frac{1}{2}} (\Delta Y_{HR} - L (X_1^k + \Delta X)) \right\|_F^2 + \gamma \|\Delta X\|_{2,1}
\]

rewritten as

\[
\Delta X^{(k+1)} = \arg \min_{\Delta X} \left\| \Lambda_{HR}^{-\frac{1}{2}} (\Delta \tilde{Y}_{HR}^k - L \Delta X) \right\|_F^2 + \gamma \|\Delta X\|_{2,1}
\]

with

- $\Delta \tilde{Y}_{HR}^k = Y_{HR} - LX_1^k$ : predicted change image at $t_2$.
- Equivalent to a spectral deblurring problem.
- Convex data-fitting term and regularization, non-smooth regularization
  → solution using proximal algorithms (i.e. forward-backward).
Multi-band optical image change detection

Robust Fusion approach

**Optimization w.r.t. $\Delta X$**

(fixing $X_1 = X_1^{(k)}$)

$$\min_{X_1, \Delta X} \left\| A_{LR}^{-\frac{1}{2}} (Y_{LR} - X_1 R) \right\|_F^2 + \left\| A_{HR}^{-\frac{1}{2}} (Y_{HR} - L (X_1 + \Delta X)) \right\|_F^2 + \mu \left\| X_1 \right\|_F^2 + \gamma \left\| \Delta X \right\|_{2,1}$$

**Optimization problem**

$$\Delta X^{(k+1)} = \arg \min_{\Delta X} \left\| A_{HR}^{-\frac{1}{2}} (\Delta Y_{HR} - L (X_1^k + \Delta X)) \right\|_F^2 + \gamma \left\| \Delta X \right\|_{2,1}$$

rewritten as

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  → solution using proximal algorithms (i.e. forward-backward).
Optimization w.r.t. $\Delta X$

(fixing $X_1 = X_1^{(k)}$)

$$
\min_{X_1, \Delta X} \left\| \Lambda_{\text{LR}}^{-\frac{1}{2}} (Y_{\text{LR}} - X_1 R) \right\|_F^2 + \left\| \Lambda_{\text{HR}}^{-\frac{1}{2}} (Y_{\text{HR}} - L (X_1 + \Delta X)) \right\|_F^2 + \mu \left\| X_1 \right\|_F^2 + \gamma \left\| \Delta X \right\|_{2,1}
$$

Optimization problem

$$
\Delta X^{(k+1)} = \arg \min_{\Delta X} \left\| \Lambda_{\text{HR}}^{-\frac{1}{2}} \left( \Delta Y_{\text{HR}} - L (X_1^k + \Delta X) \right) \right\|_F^2 + \gamma \left\| \Delta X \right\|_{2,1}
$$

rewritten as

$$
\Delta X^{(k+1)} = \arg \min_{\Delta X} \left\| \Lambda_{\text{HR}}^{-\frac{1}{2}} \left( \Delta \tilde{Y}_{\text{HR}}^k - L \Delta X \right) \right\|_F^2 + \gamma \left\| \Delta X \right\|_{2,1}
$$

with

- $\Delta \tilde{Y}_{\text{HR}}^k = Y_{\text{HR}} - L X_1^k$: predicted change image at $t_2$.
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Experiments on synthetic images
Detection performance

- Situation 1: HR-MS/LR-HS
- Situation 2: HR-PAN/LR-HS
- Situation 3: HR-PAN/LR-MS

Table: Situations 1, 2 & 3: quantitative detection performance (AUC and distance).

<table>
<thead>
<tr>
<th></th>
<th>$\hat{D}_{RF}$</th>
<th>$\hat{D}_F$</th>
<th>$\hat{D}_{WC}$</th>
<th>$\hat{D}_{DS}$</th>
<th>$\hat{D}_{SD}$</th>
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<tbody>
<tr>
<td><strong>Situation 1</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AUC</td>
<td>0.997469</td>
<td>0.981039</td>
<td>0.941408</td>
<td>0.843685</td>
<td>0.847518</td>
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<tr>
<td>Dist.</td>
<td>0.990299</td>
<td>0.951995</td>
<td>0.887789</td>
<td>0.766677</td>
<td>0.771277</td>
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<td><strong>Situation 2</strong></td>
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<td></td>
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<tr>
<td>AUC</td>
<td>0.997418</td>
<td>0.931047</td>
<td>0.89517</td>
<td>0.790859</td>
<td>0.785019</td>
</tr>
<tr>
<td>Dist.</td>
<td>0.990299</td>
<td>0.883488</td>
<td>0.833783</td>
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<td>0.712771</td>
</tr>
<tr>
<td><strong>Situation 3</strong></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>AUC</td>
<td>0.994929</td>
<td>0.94522</td>
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<td>0.779522</td>
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<tr>
<td>Dist.</td>
<td>0.991699</td>
<td>0.915992</td>
<td>0.864686</td>
<td>0.713471</td>
<td>0.706871</td>
</tr>
</tbody>
</table>
Experiments on real images
Data description

Observed image at $t_1$ (04/15/2015):
- Local: Lake-Tahoe (CA) USA.
- Sensor: Landsat 8.
- Image size: $175 \times 180$ pixels.
- Spatial resolution: 30m per pixel.
- Spectral resolution: 3 spectral bands (MS) in RBG visible spectrum.

Preprocessing:
- Manual alignment.

Observed image at $t_2$ (09/22/2015):
- Local: Lake-Tahoe (CA) USA.
- Sensor: Landsat 8.
- Image size: $350 \times 360$ pixels.
- Spatial resolution: 15m per pixel.
- Spectral resolution: PAN in RBG visible spectrum.

Compared Methods:
- Robust Fusion approach ($\hat{D}_{RF}$).
- Fusion approach ($\hat{D}_F$).
- Worst-case approach ($\hat{D}_{WC}$).
Multi-band optical image change detection

Robust Fusion approach

Experiments on real images

Visual results

Scenario $S_4$: (a) LR-MS observed image $Y_{LR}$, (b) HR-PAN observed image $Y_{HR}$, (e) change mask $\hat{D}_{WC}$ estimated by the WC approach, (d) change mask $\hat{D}_F$ estimated by the fusion approach and (c) change mask $\hat{D}_{RF}$ estimated by the proposed approach. From (f) to (h): zoomed versions of the regions delineated in red in (a)–(c).
Conclusions

Outline

Introduction

Multi-band optical image fusion
  Problem statement
  Fast fusion solving a Sylvester equation
  Experiments

Multi-band optical image change detection
  Fusion approach
  Robust Fusion approach

Conclusions
Conclusions

Fusion

- fusion of multi-band images formulated as a linear inverse problem
- spectral regularization: constraining the estimation in a lower-dimensional space
- spatial regularizations:
  - Gaussian prior
  - dictionary-based sparse prior
  - ...
- explicit solution under generalized Thinonov regularizations, which can be embedded into iterative algorithms
  - for more complex priors
  - when estimating jointly other parameters (noise variance, spectral response,...)

- assumes spectrally-invariant spatial blurs... not valid for astrophysical data
  → see [GOBD20] and [GOB^+ 20]

- fusion as a convenient framework to address change detection problems
Fusion

- fusion of multi-band images formulated as a **linear inverse problem**
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- fusion of multi-band images formulated as a \textit{linear inverse problem}
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- fusion as a convenient framework to address change detection problems
Conclusions

Change detection

Fusion approach
- unsupervised CD between optical images of different spatial/spectral resolutions
- assumes prior knowledge on the forward model (degradations)
- steps tailored by the end-user
- provides 2 CD maps of different resolutions

Robust fusion approach
- unsupervised CD between optical images of different spatial/spectral resolutions
- assumes prior knowledge on the forward model (degradations)
- estimates high resolution latent images and CD images
- provides 1 CD map of high spatial and spectral resolutions

For images of different modalities (e.g., optical and SAR)
- no forward model available
- no physically interpretable latent space
- latent space identified by data-driven methods, e.g., dictionary learning [FDC+19]
Conclusions

Change detection

Fusion approach
- unsupervised CD between optical images of different spatial/spectral resolutions
- assumes prior knowledge on the forward model (degradations)
- steps tailored by the end-user
- provides 2 CD maps of different resolutions

Robust fusion approach
- unsupervised CD between optical images of different spatial/spectral resolutions
- assumes prior knowledge on the forward model (degradations)
- estimates high resolution latent images and CD images
- provides 1 CD map of high spatial and spectral resolutions

For images of different modalities (e.g., optical and SAR)
- no forward model available
- no physically interpretable latent space
- latent space identified by data-driven methods, e.g., dictionary learning [FDC+19]
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Multi-band optical imaging
From fusion to change detection

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Joint work with Q. Wei, V. Ferraris, J.-Y. Tourneret and M. Chabert

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